APPLICATION OF MARKOV CHAIN MODEL IN TRANSITION OF GIRLS IN PUBLIC SECONDARY SCHOOLS; A case study of Kisumu West Sub-county, Kenya

BY

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A project report submitted in partial fulfilment of the requirements for the degree of Master of Science in Applied Statistics

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Declaration

This project is my own work and has not been presented for a degree award in any other institution.

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ACKNOWLEDGEMENT

I wish to express my sincere and gratitude acknowledgement to my supervisor Prof. Onyango Fredrick and Dr. Edgar Otumba for their guidance and steadfast support throughout the writing of this project. I am also grateful to Donnie Munyao who has made me understand LaTeX conversion and other members of staff of the department Applied Statistics and Actuarial science Maseno University for taking me through the course. I am also grateful to Mr. Kamguna, the District Education Officer Kisumu West sub-county and the entire staff for data provision and their unwavering support. Special thanks go to my loving wife Renalda Awino and son Markov Will not forgetting my beloved grandmother Priscah Magige and brothers Kennedy Ochieng and Zachary Onditi for their continued moral, material and spiritual support for the success of this study. Above all, I am thankful to the Almighty God for giving me health, wisdom and abundant knowledge to undertake this course. May God bless every hand that contributed to the success of this work in Jesus name!

DEDICATION

This work is sincerely dedicated to the Almighty God that is helping me do this work according to His loving kindness. Glory be to Him. Shalom!

ABSTRACT

Transition is a very important factor in determining success of a student in secondary school. It forms a basis for implementing poverty reduction strategy by the government through acquisition of basic literal skills to increase opportunities for employment. Girls have not had an efficient transition from form one to the next form in secondary schools because of choices of secondary school types they enroll in. This research uses a Markov chain model to compare and predict the percentage transition rates, expected duration of study and absorbing rates from form one to form four in a period of four years between 2012-2015 of girls in pure public secondary schools compared to those in mixed secondary schools in Kisumu west Sub county of Kisumu county Kenya. The objectives of this study were to determine and compare the sub county's transition rates, expected durations of study, absorption rates and recommend the best school that will favour efficient transition for girls. All the four pure girls secondary schools in the sub county were used and equivalent mixed schools picked by purposive random sampling. The cohort for the study included the sub county's form one girl from pure girl secondary and girls from mixed secondary schools enrolled in 2012, monitored up to form four in 2015. The study realized that transition rates for girls in pure girls' schools were higher than those in mixed secondary schools, expected duration study was shorter in mixed schools compared to those in pure girls schools and finally higher absorption rates of girls in pure compared to those in mixed secondary schools in Kisumu West sub county. The dropout rate was higher for girls in mixed but highest in form threes in mixed. The study would help in choosing the effective school type for girls and stake holders in adopting the appropriate way to curb girls attrition.

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ABBREVIATIONS AND ACRONYMS

BOM:	Board of Management
EFA:	Education for All
FSE:	Free Secondary education
MOEST:	Ministry of Education
H.I.V:	Human Immune Virus
AIDS:	Acquired Immune Deficiency Syndrome
K.C.S.E:	Kenya Certificate Of Secondary Education
DTMC:	Discrete Time Markov Chain

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Chapter 1

INTRODUCTION

1.1 Background of the Study

Education is a process or an art of imparting knowledge, skills and judgement. It is categorized in three levels; primary, secondary and tertiary. For an individual to complete these three levels of education, transition from one level to another is necessary starting with primary as a foundation followed by secondary which the government of Kenya considers a basic and compulsory for all its citizens. Transition within each level should be considered so that a student takes the least time possible for maximum achievement of set academic goals and objectives. Secondary education provides an important link between basic education and tertiary education that enables man to maximally exploit natural resources [3] It is therefore a section of education that prepares human capital development and provision of life opportunities for the realization of quality and equity education.

Students especially the girl child enrolment, the transition from one form to another, form repetition and completion within the stipulated four year period needs to be monitored for an effective planning and proper decision making for human capital development.

Education system is comparable to a hierarchical organization in which after an academic year, four possibilities arise in the new status of the student; The student may move to the next higher form, may repeat the same form, may leave the system successfully as graduate or dropout of the system before attaining the maximum qualification (see [23, 24]). It is estimated that an average earning of an individual increases by 11% within each additional year of education and each additional year of material education reduces childhood mortality by 8% (see [27])

Since the initiation of free primary education (FPE) by the government in 2003, there has been a high expectation that the enrolment and transition rates of girls in secondary would increase however, this has not been realized. The issue has brought a lot of concern from parents, teachers, education sectors and sponsors why the reverse is taking place. This has brought the attention of the actual picture of what's happening at every level in order to find out an effective decision making for transition of girls by Markov Chain.

A Markov chain is a stochastic process with the Markov property; given the present state, the future and the past are independent. This means that, the next (form) state of a student depends only on the present state (form) of the student. The study applied a Markov chain model of transition to show probabilities of students moving to the next states from current states. This showed their transitions, expected durations of study and absorbing rates of girls in pure public girls schools compared to those in mixed secondary schools in Kisumu West sub county of Kisumu County Kenya.

In this study, form 4 graduation was considered as a successful completion of secondary education level having done KCSE examination. The number of girls who graduated successfully are considered as survival cases and those who failed are considered as attrition cases.

1.2 Statement of the problem

Transition, retention and completion rates of girls in public mixed secondary schools is of a great concern because it remains far much below the admission or enrolment level. The national completion rate in 2004 was 87.5% for girls and 92.3% for boys reported. Introduction of Free Secondary Education (FSE) realized an increased enrolment in secondary schools for both boys and girls however, transition, retention and completion rate for girls are still low. This research therefore investigated and compared transition, retention and completion rates of girls in mixed secondary schools compared to pure girls secondary schools in Kisumu West Sub-County by Markov Chain model.

1.3 Main Objective of the Study

The main objective of the study was to determine the transition rates of girls in pure girls secondary schools compared to those in mixed secondary schools in Kisumu west sub county of Kisumu county Kenya.

1.4 Specific Objectives of the study

This study was guided by the following specific objectives;

1. To compare the expected duration of study of girls in mixed secondary schools compared to those in pure girls secondary schools in Kisumu West Sub County;
2. to compare the absorbing rates of girls in mixed secondary schools compared to those in pure girls schools in Kisumu West Sub County;
3. to recommend the suitable school for girls' education with efficient transition and the shortest expected duration of study.

1.5 Research Hypothesis

H_{0_1} There is no difference between girls' transition in mixed secondary school to those in pure girls' secondary school and in Kisumu west Sub County.

H_{0_2} There is no difference between girls' expected duration of study in mixed secondary school to those in pure girls' secondary school in Kisumu west Sub County.

H_{0_3} There is no difference between absorbing rates of girls in mixed secondary school compared to those in pure girls secondary school in Kisumu west Sub County.

1.6 Significance of the study

Education is an important factor in determining the future success of an individual in the society. Secondary education requires at least four years for completion. The purpose of this study was to monitor the transition rates, expected duration of study and absorption rates of girls in mixed secondary schools in comparison to those in pure girls secondary schools in Kisumu west by Markov Chain. The results of this study shall be used to advice school managers, sponsors, parents and teachers on a secondary school type that favours girls transition to academic success.

1.7 Assumptions of the Study

The following assumptions were taken into consideration for the model to be appropriate;

- The dropouts are assumed be uniformly distributed in the period $(x; x+1)$ where x is the year of study
- Admissions took place only in form one.
- Population assumed closed; no migration and immigration to neighboring sub counties.
- No form repetition.

1.8 Basic concepts

This study employed Markov Chain, which is a stochastic model. It is where the outcomes of the previous experiments do not influence the outcomes of the next experiments [7] Martin L. Puterman defines it as; Let $\{x_1, x_2, \ldots\}$ be a sequence of random variables which assume values in discrete (finite or countable) state space S. We say that $\{X_t, t = 0, 1, 2, \ldots\}$ is a markov chain if

$$P_{ij} = Pr(X_t = j | X_{t-1} = i) \text{ where } i, j = 0, 1, 2, \ldots, n.$$

That is, the process starts in state one (i) and move successfully to the next state (j) on a state space. It's called state space; a set of all possible states of dynamic system and each move on a state space is called *a step*. The sequence of random variables $\{X_1, X_2, X_3, \ldots, X_n\}$ has a Markov property

1.8.1 First Order Markov Chain

A Markov chain of first order is where the current state is solemnly dependent on the immediate previous state and the chances that a process is in state 'i' at time $(t-1)$ is presented as;

$$P_{ij} = Pr(X_t = j | X_{t-1} = i) \text{ where } P_{ij} \text{ can be estimated as}$$

$$P_{ij} = \frac{q_{ij}}{\sum_{j=1}^{n} q_{ij}}$$

where $i, j = 0, 1, \ldots, n, q_{ij}$ is the historical frequency of transition from"i" to state 'j' and n is the maximum number of states.

1.8.2 Second and Higher order Markov chain

A Markov Chain is termed as second or higher order Markov Chain if m is greater than one and the following conditions are satisfied;

$$P(i_2, i_1, j, t) = Pr(X_t = j|X_{t-1} = i_1, X_{t-2} = i_2, \ldots, X_0 = i_0) \tag{1.1}$$

$$= Pr(X_t = j|X_{t-1} = i_j, X_{t-2} = i_2) \tag{1.2}$$

The equations 1.1 and 1.2 are referred to as second and higher order respectively. The equation

$$P(X_i|X_{i-1}, X_{i-2}, \ldots, X_1) = Pr(X_{t-1}, \ldots, X_{t-m})$$

where the future states depend on the past m states is an m higher order Markov chain. Markov chain of first order, states the transition probabilities at each time or step hence sufficient for this model.

Equally the model was used to predict number of dropouts by girls at each class of study which will assist interested parties to make informed decision about secondary education management in view to curb dropouts. Since the initiation of Free Secondary Education (FSE), there is need to realize efficient girl child transition than what is currently being financed to meet the future academic goal and objective of the country using Kisumu West Sub County secondary school enrolment data.

Chapter 2

LITERATURE REVIEW

Over the years, many techniques have been suggested for forecasting enrolment and students flow at any level in education systems. Wing [38] classified them into curve fitting, causal models, attitude surveys and judgemental techniques. Healey et al [14] classified them into the judgemental, Markov process, trend analysis, regression, simulation and the ratio techniques.

A Markov chain model is a stochastic model with random events which have been used in diverse fields such as computer science, engineering, mathematics, genetics, agriculture economics, education, biology, etc. (see [15, 17, 34]).

Markov chain has been widely used to model stochastic processes and to evaluate time to event data. Musiga et al [23] modelled a hierarchical system with a single absorbing state for an education system where drop-outs and graduates were grouped together. Later, Musiga et al [24], modelled a hierarchical system with double absorbing states for an education system, where graduates were separated from dropouts. These two papers form the basis for this study. In both, the education system under study is narrowed to one institution but this study generalizes to the groups of institutions in a sub county to be specific and takes into consideration girl child disaggregation of the data.

Generally, most of these studies have been carried out in primary level of study, secondary school levels, universities and their respective inter-institutional transition levels of learning. Nyandwaki [27] published a research on application of Markov chain model in studying progression of secondary school students by sex during the introduction of free secondary education: a case study of Kisii central district. In his study he compared the transition of girls to boys in secondary schools. He came up with the following conclusions from his discussions; The transition and completion rates for male students are higher than that of female students in Kisii Central District. The expected duration of schooling was higher for male students compared to the female students. Female students were found to have lower expectation of schooling than their male counterparts. The dropout rates for female students was higher than that of male students in Kisii Central District and finally the transition rates from Form 1 to Form 2 was higher for female students than that of male students. In all other Forms, transition rate for the male students was higher. Form three had the least retention rate in the District for both sexes.

Education is one of the basic services offered by governments and other stakeholders to society. Bray [6] posted that education is a tool for economic development and therefore there is need to constantly review inputs and outputs in the system. If education is to meet the goal of economic development in any nation, then aspects of internal efficiency should be examined critically. The term ‘efficiency’ as applied in education refers to the extent to which education yields desirable results to the society and individuals Sang et al [32].

A study that was carried out by the Association for the Development of Education in Africa [1], indicated that there were a number of obstacles to high level achievement of internal efficiency in schools. It was noted that delays in the acquisition of inputs to a large degree affected internal efficiency. For instance, even after the resources were made available, some schools would still take two to three months to avail to the students. Of concern also was the increased student numbers leading to shortages of teachers. This problem according to the study was further enhanced by the fact that governments have been replacing only those teachers who die, resign or retire. The ability of most governments to maintain positive trends in education is held back in particular by severe budgetary constraints. Eisenmon [11] in a study on wastages in secondary education reported that dropout rates in developing countries often are quite high. Besides, the study found that Boarding Schools were preferred by most stakeholders since students did better than their day schools counterparts in national examinations. Notably conspicuous in this study was that dropout rates were higher in Day schools compared to Boarding schools. According to this study, the highest rates were in the sub-Saharan African countries where each year, about 22 percent of primary schools pupils and 21 percent of secondary schools students were dropping out of school. Out of the total percentage of dropouts in secondary education, Boarding schools accounted for 8 percent while Day schools accounted for 13 percent. The North African and Middle Eastern countries averaged about 12 percent for the primary grades and 21 percent for the secondary grades.

From these percentages, single streamed schools accounted for 7 percent while 14 percent came from schools with two streams or more. This was because the single stream schools were fewer than the double or more streamed schools and the study established that most education stakeholders preferred large size schools (see [11]).

On grounds of the economies of large-scale production, the Latin American and Caribbean countries averaged 9 percent and 8 percent for primary and secondary schools respectively. The data from East and South-East Asia were too sporadic to support meaningful averages, but the available number appeared comparable to those for Latin America. According to EFA [10], Kenya has the largest percentage of her children in both primary and secondary school of which 13 percent drop out of school at any given time due to poverty, early marriages, HIV/AIDS pandemic and poor learning environment; [28], indicates a transition rate of 70% from primary to secondary education cycle and this is an indication that approximately 30% of the pupils nationally are unable to proceed to secondary schools. A study on rural day schools by Ncube [25] in Zimbabwe found that the number of students dropping from a level increased with the level of schooling.

The study aimed at comparing the transition rates, expected duration of study and absorbing rates of girls in mixed secondary schools to those in pure girls' secondary schools in Kisumu west Sub County of Kisumu County. It is relevant since enrollment to completion ratio of girls in secondary schools is wanting. There is need for girls to be allowed to study in a school type that will favour their effective transition and completions to enable them compete with male cohorts.

Chapter 3

RESEARCH METHODOLOGY

3.1 Introduction

This chapter presents research design, location of the study, target population, sampling method and model development. The study applied Markov chain model to compute transition rates, expected duration of study and absorbing rates of girls in mixed secondary schools to those in pure girls' secondary schools in Kisumu west sub County of Kisumu County.

3.2 Research Design

The research used purposive sampling where the population was divided into groups of girls from pure girls and girls from mixed secondary schools in Kisumu west Sub County. All the pure girls schools were used, mixed schools were picked based on population and geographical distribution within the sub county.

3.3 Location of the Study

The study was carried out in Kisumu west Sub County, one of the sub counties bordered by Kisumu East to the eastern part, Seme Sub County to the west, Maseno and Vihiga to the north and by Lake Victoria to the South. According to the population projection from Kenya Bureau of Statistics, it has a population of 131246 people in 2016. It covers an area of 212.9 square kilometers. It's a cosmopolitan with the major tribe being Luos and few Luhyas to the west. The major occupation of those to the southern is fishing since large percentages of the schools are bordered by Lake Victoria. People to the Western, Eastern and Northern regions are of dynamic economic activities such as farming and business. The area was chosen because of its cosmopolitan nature, being that it has schools in both rural and urban areas with activities that can influence transition expected duration of study and absorption rates, a presentation of a normal population.

3.4 Target population

The target populations for the study were girls from both pure public girls and mixed secondary schools enrolled in 2012 with their transitions monitored up to form four in 2015 in Kisumu West sub County.

3.5 The study Sample

The study applied purposive sampling technique where population was divided in two main groups; girls from pure public girls' schools and girls from public mixed schools. All the pure girls schools were used and mixed schools were chosen based on shared attributes or characteristics, geographical distribution and school category. The data used was a secondary data of form ones enrolled in 2012 which had a population of 976/1200; a representation of 81.333 percent of the total enrolment with 95 percent confidence level with a Z-value of 1.96 precision of 0.05 absolute error.

3.6 Model Development

3.6.1 Form $P_{i,j}$

The variable $P_{i,j}$ represented form transition in Kenyan secondary system of education. In this study, a Markov Chain Model on transition was used to compare the transition of girls in mixed secondary schools to those in pure public girls' secondary schools in Kisumu West sub County of Kisumu County. Markov chain is a process with a finite number of states or outcomes, or events in which the probability of being in a particular state at step $n+1$ depends only on the state occupied at step n.

Let $S = \{S_1, S_2, \ldots, S_r\}$ be the possible states and

$$P_n = \begin{pmatrix} P_1 \\ P_2 \\ \vdots \\ P_r \end{pmatrix}$$

The vector of probabilities of each state at step n that is, the i^{th} entry of P_n is the probability that the process is in state S_i at step n. For such a probability vector,

$$P_1 + P_2 + \cdots + P_r = 1$$

Let

$$P_{ij} = Pr(\text{State at step } (n+1) = S_j/\text{State at step } n = S_i)$$

A DTMC is referred to as homogeneous if and only if its transition probabilities do not depend on time n such that;

$$P(X_{n+1} = j|X_n = i) = P(X_1 = j|X_0 = i)$$

A homogeneous DTMC is described by its transition matrix $P = [P_{i,j}], i, j \in E$. $P_{i,j}$ is the transition probability; probability of making movement from state i to state j. In the transition probability matrix, $P_{i,j}$ where i is the initial state (form) and j is the next state (form) i.e.,

P_{12}: Probabaility of a form 1moving to form 2
P_{23}: Probabaility of a form 2 moving to form 3
P_{34}: Probabaility of a form 3 moving to form 4
P_{45}: Probabaility of a form 4 graduating

The canonical form of transitional matrix is;

$$\begin{pmatrix} P_{11} & P_{12} & \cdots & P_{1r} \\ P_{21} & P_{22} & \cdots & P_{2r} \\ \vdots & \cdots & \cdots & \vdots \\ P_{r1} & P_{r2} & \cdots & P_{rr} \end{pmatrix}$$

P_{ij} is the (conditional) probability of being in state S_i at step $n+1$ given that the process was in state S_j at step n. P is called the transition matrix. P contains all the conditional probabilities of the Markov chain. It can be useful to label the rows and columns of P with the states as bellow;

$$\text{State } n+1 \begin{cases} S_1 \\ S_2 \\ S_3 \\ S_4 \end{cases} \overbrace{\begin{pmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{pmatrix}}^{\text{State } n\text{; where } n=4}$$

In this research P_{11}, P_{22}, P_{33} and P_{44} represent transition probabilistic matrix element for the four year duration; 2012-2015.

3.6.2 Graphical presentation of a DTMC

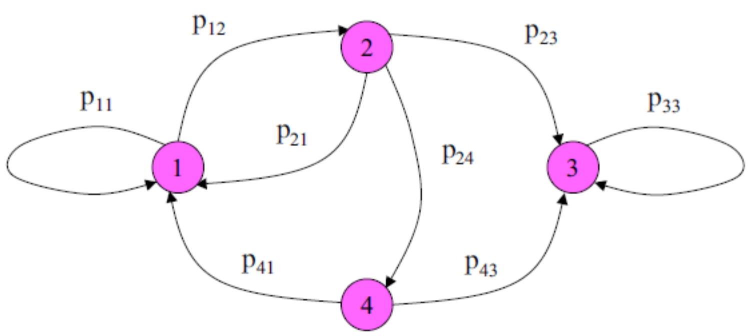

Figure 3.1: A graphical presentation of a DTMC

3.6.3 Expected Duration of Study

The expected duration of study was obtained by the formula $(1-Q)^{-1}(1111)^T$ according to Beck and Pauker (1983) where; I is the identity matrix, Q is the transitional matrix and

$$(1111)^T = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

3.6.4 Absorbing Markov Chain

An absorbing state is a state which becomes permanent once it has been entered thus the transition probabilities between absorbing states is represented by one, justifying the use of the identity matrix. Transition from an absorbing state to a non-absorbing state which is impossible, is represented by zero, hence the matrix of zeroes. Transitions from non-absorbing states to absorbing states are possible, likewise between non-absorbing states.

In the mathematical theory of probability, an absorbing Markov chain is a Markov chain in which every state can reach an absorbing state. An absorbing state is a state that, once entered, cannot be left. Like general Markov chains, there can be continuous-time absorbing Markov chains with an infinite state space. However, this article concentrates on the discrete state-space case [37].

A Markov chain, which is not absorbing, is called transient; a state i is said to *transient* if, given that we start in state i there is a non zero probability that we will never return to that state. If we have an absorbing Markov chain with t transient states and r absorbing states, the transition probability matrix P, will take the following canonical form

$$P = \begin{pmatrix} Q & R \\ O & I \end{pmatrix}$$

Where; Q is a $t \times t$ matrix, q_{ij} being the probability that a student who is in class i at time $t - i$ will be in class j at time $t; i, j = 1, 2, \ldots, t$,

R is a non-zero $t \times r$ matrix, r_{ik} being the probability that a student in class i at time $(t - 1)$ will graduate with final education k at time $t; i = 1, 2, \ldots, t$ and $k = 1, 2, \ldots, r$;

O is an $r \times t$ zero matrix and I is an $r \times r$ identity matrix.

The first t states are transient states and the last r states are absorbing states (see [4]). The ij^{th} entry, of the matrix P_{ij} gives the probability that the Markov chain, starting in state S_i will be in state S_j after n steps by Theorem 3.6.5.1. The canonical form of the matrix P_n is given as;

$$P^n = \begin{pmatrix} Q^n & R^n \\ O & I \end{pmatrix}$$

Where;

Q^n is a $t \times t$ matrix which gives the probability that a student who is in class i will be in class j, n years later, for i and $j = 1, 2, \ldots, t$,

$R^n = (I+Q+Q^2+\cdots+Q^{n-1})R$ is a $t\times r$ matrix which gives the probability that a student who is in class i will graduate with final education k within n years, $i = 1, 2, \ldots, t; k = 1, 2, \ldots, r$. It is also called the completion rate; O is an $r \times t$ matrix of zeros which gives transition.

Probabilities from absorbing states to non-absorbing states in n steps and, I is a $r \times r$ identity matrix which gives transition probabilities between absorbing states in n steps.

Urakabe et al, [36] and Silverstein et al, [33] concluded that while the probability matrix summarizes transition probability of the cohort, the transient states analysis allows prediction or prognosis for an individual subject, given their starting state, current state and cycle.

3.6.5 n-step Transition Matrix

The n-step transition probability matrix takes the canonical form of chapman-kolmogorov equation stated below:

Theorem 3.6.5.1 (Chapman-Kolmogorov Equation). *Let* $P^n_{i,j} = P(X_{m+n} = j|X_0 = i)$ *be the probability that an element is in state* j *after* $(m+n)$ *steps from state* i*. Then*

$$P^{n+m}_{i,j} = \sum_{k=0}^{\infty} P^n_{i,k} P^m_{k,j},\ n, m \geq 0 \text{ for all } i, j.$$

3.7 The fundamental Matrix solution

Matrix solution provides an exact solution of the time spent in each state, conditional on the entry state in which an individual enters the model. Matrix solution is restricted to time homogeneous Markov chains. The transition probability matrix of a chain that contains absorbing states is divided into four sections: Q contains transition probabilities between transient states; R contains transition probabilities, from transient to absorbing states; O is a zero matrix, and I is an identity matrix (see [4, 14]).

		To: Transient States	To: Absorbing States
From:	Transient States	Q	R
	Absorbing States	O	I

Figure 3.2: Fundamental Matrix Solution

Separation of a probability transition matrix containing absorbing states into 4 components: Q contains transition probabilities between transient states; R contains transition probabilities from transient to absorbing states; O is a zero matrix, and I is an identity matrix.

The average number of cycles that a subject resides in transient states before absorption, given a specified starting state, is estimated from the fundamental (N) matrix. Calculating N is the matrix algebraic equivalent of taking the inverse of the transition probabilities in Q ([4, 14]).

The N matrix specifies the average number of cycles that a subject resides in transient states such that; $N = (I - Q) - 1$ where I is an identity matrix and Q is the square matrix of the transient probabilities within P.

Multiplication of the number of cycles by the length of the cycle gives the expected duration in each state, conditional on a starting state. The sum of these durations gives an estimate of expected survival, conditional on a starting state (see [4]). The variance of N is given by the V matrix with $V = N(2n - I)$ where n is a copy of N with only the diagonal entries preserved (and zeroes elsewhere) and is a matrix with each entry of N squared (see [4, 33]). Each element of V represents the variance of the corresponding element of N. The square root of each element of V is the standard deviation of the corresponding element of N.

For an absorbing Markov chain, the matrix N is the fundamental matrix where;

$$N = (I - Q) - 1 = I + Q + Q^2 + \cdots$$

Where the ij^{th} entry n_{ij} of the matrix N is the expected number of times the process is in the transient state S_j given that it started in the transient state S_i. Hence N gives the average number of cycles that a subject resides in transient states before absorption, given a specified starting state. The states of the education system were denoted by integers $1, 2, \ldots, n$ at time $t = 0, 1, 2, \ldots$ while P_{ij} denoted the probability that a student in class i at time $t-1$ will be in class j at time t, then the transition matrix

$$P = ((P_{ij})); Ij = 1, 2, \ldots, n.$$

The non-absorbing states (transient states) were four and they were represented by values 1, 2, 3, and 4. This implies that the Q component of the transition matrix P is a $4{\times}4$ matrix. The number of absorbing states were two and they were represented by values 5 and 6. The absorbing state 5 represents graduation from the system after attaining the maximum qualification and state 6 represents dropping out of the system before attaining the maximum qualification. Hence the R component of matrix P was a $4{\times}2$ matrix.

According to Beck and Pauker, [4], Brown and Brown, [14], the purpose of the transition matrix is to represent the probability of movement between states in a single time period. In this case, it was the probability that a student will reach a particular state by the end of the year of study.

Chapter 4

MODEL FITTING

4.1 Initial transition Probabilities

By letting $n_{ij}(t)$ represent the number of students in class i at time $(t-1)$ who will be in class j at time t, and $n_i(t-1)$ to represent the number of students in class i at time $(t-1)$, and by assuming the multinomial distribution, the transition probabilities can be estimated by;

$$P_{i,j} = \frac{n_{ij}(t)}{n_i(t-1)}, i, j = 1, 2, \ldots, N$$

P_{ij} is the proportions of students who are in class i at time $(t-1)$ who ends up being in class j at time t.

4.2 Important Notations

Some of the notations used in the subsequent sections are defined below;

P_s =Sub county transition probability matrix,

P_{pg} =Sub county pure girls' transition probability matrix, and

P_{sm} =Sub county girls in mixed school transition probability matrix,

Q =A component of matrix P whose states are transient states representing the proportions of students who proceed to their respective next states; years 2013, 2014 and finally 2015

4.3 The Initial Transition Probability matrix for Girls in mixed secondary schools in Kisumu West Sub-County

From the data obtained from Kisumu West Sub County, girls enrolment in mixed secondary schools in Forms 1, 2, 3 and 4 for the year 2012 to 2015 enrolment and their respective transitions in forms 2, 3, and Form 4s who proceeded to the next state the following years, 2013 to 2015 were as shown in Table 4.1.

The dropout proportions before attaining maximum qualification for students who were in Forms 1, 2, 3 and 4 were as follows for girls in mixed secondary schools; (9/306) =

$0.029412, (20/297) = 0.067340, (32/277) = 0.115523, (5/245) = 0.020408$ respectively. The dropout for the whole period between 2012 and 2015 is (66/306)=0.215686 and the transition rate is (240/306)=0.784314 which represent proportion of students who graduated successfully after reaching Form 4. This gives rise to the R component of the matrix P_s for the sub county.

The Q component of the matrix P_{sm}, whose states are transient states, its elements represent the proportions of students who proceeded to Forms 2, 3 and 4 within the period 2012 to 2015. The proportions of students who proceeded to the next state from forms 1, 2, 3 and 4 in the year 2012, 2013, 2014 and 2015 are $(297/306) = 0.970588, (277/297) = 0.932660, (245/277) = 0.884477$ and $(240/245) = 0.979592$ respectively.

(The above transitions are summarized on tabulated table below)

	F1 Jan 2012-Nov 2012			F2 Jan 2013-Nov 2013			F3 Jan 2014-Nov 2014			F4 Jan 2015-Nov 2015		
Institution	Jan 2012	Nov 2012	Drop out rates	Jan 2013	Nov 2013	Drop out rates	Jan 2014	Nov 2014	Drop out rates	Jan 2015	Nov 2015	Drop out rates
Kanyamedha	66	63	3	63	59	4	59	52	7	52	50	2
Kuoyo	50	49	1	49	44	5	44	40	4	40	40	0
Dago Kokore	100	98	2	98	92	6	92	81	11	81	79	2
Obambo	90	87	3	87	82	5	82	72	10	72	71	1
TOTAL	306	297	9	297	277	20	277	245	32	245	240	5
D/O rates			9/306 0.029412			20/297 0.0673401			32/2778 0.115523			5/245 0.020408
Transition			0.970588			0.932660			0.884477			0.979592

Table 4.1: Transitions of girls in public mixed secondary schools in Kisumu west sub-county, Kenya

Thus, assuming time homogeneity, the sub county transition probability matrix

$$P_{sm}\begin{pmatrix} 0.000000 & 0.970588 & 0.000000 & 0.000000 & 0.000000 & 0.029412 \\ 0.000000 & 0.000000 & 0.932660 & 0.000000 & 0.000000 & 0.067340 \\ 0.000000 & 0.000000 & 0.000000 & 0.884477 & 0.000000 & 0.115523 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.979592 & 0.020408 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 & 1.000000 & 0.000000 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{pmatrix}$$

The Q component of the transition matrix is

$$Q_{sm} = \begin{pmatrix} 0.000000 & 0.970588 & 0.000000 & 0.000000 \\ 0.000000 & 0.000000 & 0.932660 & 0.000000 \\ 0.000000 & 0.000000 & 0.000000 & 0.884477 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 \end{pmatrix}$$

The R Components of the matrix R_{sm} is

$$R_{sm} = \begin{pmatrix} 0.000000 & 0.029412 \\ 0.000000 & 0.067340 \\ 0.000000 & 0.115523 \\ 0.979592 & 0.020408 \end{pmatrix}$$

4.4 Initial Transition Probability for Girls in Pure Girls secondary schools in Kisumu West Sub County

In contrast, data obtained from Kisumu West Sub County's girls enrollment in pure girls' secondary schools in Forms 1, 2, 3 and 4 for the year 2012 to 2015 their respective transition in forms 1, 2, 3, and 4 who proceeded to the next state the following years, 2013 to 2015 were as shown in Table ii below. The dropout proportions before attaining maximum qualification for students who were in Forms 1, 2, 3 and 4 were also shown as follows. $(3/670) = 0.0044778, (9/667) = 0.013493, (13/658) = 0.019757, (2/645) = 0.003101$. The dropout for the whole period between 2012 and 2015 was $(27/670) = 0.040299$ and the transition rate is $(643/670) = 0.959701$ which represent proportion of students who graduated successfully after reaching Form 4. This gave rise to the non absorbing (R) component of the matrix P_s for the sub county.

The Q component of the matrix P_{pg}, whose states are transient states, its elements represent the proportions of students who proceeded to Forms 2, 3 and 4 within the period 2012 to 2015. The proportions of students who proceeded to the next state from forms 1, 2, 3 and 4 in the year 2013, 2014 and 2015 were; $(667/670) = 0.995522, (658/667) = 0.986507, (645/658) = 0.980243 and (643/645) = 0.996899$ respectively.

(The above transitions are summarized on tabulated below)

	F1 Jan 2012-Nov 2012			F2 Jan 2013-Nov 2013			F3 Jan 2014-Nov 2014			F4 Jan 2015-Nov 2015		
Institution	Jan 2012	Nov 2012	Drop out rates	Jan 2013	Nov 2013	Drop out rates	Jan 2014	Nov 2014	Drop out rates	Jan 2015	Nov 2015	Drop out rates
A.I.C Alara	60	59	1	59	56	3	56	52	4	52	52	0
Ojolla Girls	200	200	0	200	198	2	198	195	3	195	194	1
Huma Girls	170	168	2	168	166	2	166	162	4	162	161	1
Sinyolo Girls	240	240	0	240	238	2	238	236	2	236	236	0
TOTAL	670	667	3	667	658	9	658	645	13	645	643	2
D/O rates			3/670 0.004478			9/667 0.013493			13/658 0.019757			2/645 0.003101
Transition			0.995522			0.986507			0.980243			0.996899

Table 4.2: Transition of girls in public pure girls secondary schools in kisumu west sub county, kenya

Thus, assuming time homogeneity, the girls' transition in pure girls' school in the sub county has the probability matrix P_{pg} shown.

$$P_{pg} = \begin{pmatrix} 0.000000 & 0.995522 & 0.000000 & 0.000000 & 0.000000 & 0.004478 \\ 0.000000 & 0.000000 & 0.986507 & 0.000000 & 0.000000 & 0.013493 \\ 0.000000 & 0.000000 & 0.000000 & 0.980243 & 0.000000 & 0.019757 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.996899 & 0.003101 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 & 1.000000 & 0.000000 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{pmatrix}$$

The absorbing (Q) component of the transition matrix is

$$Q_{pg} = \begin{pmatrix} 0.000000 & 0.995522 & 0.000000 & 0.000000 \\ 0.000000 & 0.000000 & 0.986507 & 0.000000 \\ 0.000000 & 0.000000 & 0.000000 & 0.980243 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 \end{pmatrix}$$

The non-absorbing (R) Components of the matrix R_{pg} is

$$R_{pg} = \begin{pmatrix} 0.000000 & 0.004478 \\ 0.000000 & 0.013493 \\ 0.000000 & 0.019757 \\ 0.996899 & 0.003101 \end{pmatrix}$$

4.5 Transition rates

Students in Form 4 were grouped into those who dropped out of the system before attaining the maximum qualification and those who actually graduated from the system. The completion rate is the $(i,k)^{\text{th}}$ element of $(I+Q+Q^2+\cdots+Q^{(x-1)})R$ Within the first year and subsequent, that is in the year $2017, 2018, \ldots$ the Sub County transition rate will be as shown below depending on the school type.

4.5.1 Transition rates for Girls in Mixed Secondary Schools in Kisumu West Sub County

The transition rates of girls in pure public girls schools in Kisumu West sub-county can be obtained as shown below;

After one year, the transition rate will be; $(I+Q)_{sm}R_{sm}$

$$\begin{pmatrix} 1.000000 & 0.970588 & 0.000000 & 0.000000 \\ 0.000000 & 1.000000 & 0.932660 & 0.000000 \\ 0.000000 & 0.000000 & 1.000000 & 0.884477 \\ 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{pmatrix} \begin{pmatrix} 0.000000 & 0.029412 \\ 0.000000 & 0.067340 \\ 0.000000 & 0.115523 \\ 0.979592 & 0.020408 \end{pmatrix} = \begin{pmatrix} 0.000000 & 0.094771 \\ 0.000000 & 0.175084 \\ 0.866427 & 0.133573 \\ 0.979592 & 0.020408 \end{pmatrix}$$

Within two years, transition rate of girls in mixed secondary schools within the sub county will determine by $(I + Q + Q^2)_{sm} R_{sm}$;

$$\begin{pmatrix} 1.000000 & 0.970588 & 0.905229 & 0.000000 \\ 0.000000 & 1.000000 & 0.932660 & 0.824916 \\ 0.000000 & 0.000000 & 1.000000 & 0.884477 \\ 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{pmatrix} \begin{pmatrix} 0.000000 & 0.029412 \\ 0.000000 & 0.067340 \\ 0.000000 & 0.115523 \\ 0.979592 & 0.020408 \end{pmatrix} = \begin{pmatrix} 0.000000 & 0.199346 \\ 0.808081 & 0.191919 \\ 0.866427 & 0.133573 \\ 0.979592 & 0.020408 \end{pmatrix}$$

The transition rates within four years using the absorbing states model from the Table 4.1 above. From the table, it is clear that by 2017 (one year later), 9.4771% of girl students who were in Form 1 in the year 2012 shall have dropped out of the system before attaining the maximum qualification while 90.5229% will transit to form two. For girls in Form 2, 17.5084% are expected to drop while 82.4916% of the students are expected to transit to form 3 after attaining maximum qualification. For those in Form 3, 86.6427% of girls are expected to transit to form four with 13.3573% drop out and finally 97.9592% are expected to successfully graduate from form four.

Two years later (2018) , the transition rates using the absorbing states model for forms 1,2,3 and 4 are likely to be; 80.0654%, 80.8081%, 86.6427% and 97.9592% respectively.

Three years later the transition rates can be obtained by $(I + Q + Q^2 + Q^3)_{sm} R_{sm}$

$$\begin{pmatrix} 1.000000 & 0.970588 & 0.905229 & 0.800654 \\ 0.000000 & 1.000000 & 0.932660 & 0.824916 \\ 0.000000 & 0.000000 & 1.000000 & 0.884477 \\ 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{pmatrix} \begin{pmatrix} 0.000000 & 0.029412 \\ 0.000000 & 0.067340 \\ 0.000000 & 0.115523 \\ 0.979592 & 0.020408 \end{pmatrix} = \begin{pmatrix} 0.784314 & 0.215686 \\ 0.808081 & 0.191919 \\ 0.866427 & 0.133573 \\ 0.979592 & 0.020408 \end{pmatrix}$$

Three years later (2019) , the transition rates using the absorbing states model for forms 1,2,3 and 4 are likely to be; 78.4314%, 80.8081%, 86.6427% and 97.9592% respectively.

4.5.2 Transition rates for Girls in Pure girls Schools in Kisumu West Sub County

The transition rates of girls in pure public schools in Kisumu West sub-county can be obtained as;

After one year, the transition rate will be; $(I+Q)_{pg}R_{pg}$

$$\begin{pmatrix} 1.000000 & 0.995522 & 0.000000 & 0.000000 \\ 0.000000 & 1.000000 & 0.986507 & 0.000000 \\ 0.000000 & 0.000000 & 1.000000 & 0.980243 \\ 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{pmatrix} \begin{pmatrix} 0.000000 & 0.004478 \\ 0.000000 & 0.013493 \\ 0.000000 & 0.019757 \\ 0.996899 & 0.003101 \end{pmatrix} = \begin{pmatrix} 0.000000 & 0.017911 \\ 0.000000 & 0.032983 \\ 0.977203 & 0.022797 \\ 0.996899 & 0.003101 \end{pmatrix}$$

One year later (2017), the transition rates for girls in pure girls schools using the absorbing states model for forms 1,2,3 and 4 are likely to be; 98.2089%, 96.7017%, 97.7203% and 99.6899% respectively

After two years, the transition rates of girls in pure public girl's school will be obtained by $(I+Q+Q^2)_{pg}R_{pg}$

$$\begin{pmatrix} 1.000000 & 0.995522 & 0.982089 & 0.000000 \\ 0.000000 & 1.000000 & 0.986507 & 0.967017 \\ 0.000000 & 0.000000 & 1.000000 & 0.980243 \\ 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{pmatrix} \begin{pmatrix} 0.000000 & 0.004478 \\ 0.000000 & 0.013493 \\ 0.000000 & 0.019757 \\ 0.996899 & 0.003101 \end{pmatrix} = \begin{pmatrix} 0.000000 & 0.181620 \\ 0.964018 & 0.035982 \\ 0.977203 & 0.022797 \\ 0.996899 & 0.003101 \end{pmatrix}$$

The transition rates two years later using the absorbing states model is as in the table above as 81.838% of the girls in form one transited to form two, 96.4018% in form two girls transited to form three, 97.7203% of form three girls transited to form four. Finally 99.6899% are expected to graduate from the system after attaining maximum qualification by the end of the fourth year of study.

Subsequently, transition after three years of study can be attained by the absorbing states using the formula; $(I + Q + Q^2 + Q^3)_{pg} R_{pg}$

$$\begin{pmatrix} 1.000000 & 0.995522 & 0.982089 & 0.962686 \\ 0.000000 & 1.000000 & 0.986507 & 0.967017 \\ 0.000000 & 0.000000 & 1.000000 & 0.980243 \\ 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{pmatrix} \begin{pmatrix} 0.000000 & 0.004478 \\ 0.000000 & 0.013493 \\ 0.000000 & 0.019757 \\ 0.996899 & 0.003101 \end{pmatrix} = \begin{pmatrix} 0.0.959701 & 0.040299 \\ 0.964018 & 0.035982 \\ 0.977203 & 0.022797 \\ 0.996899 & 0.003101 \end{pmatrix}$$

Transition rates by absorbing Markov chain can be obtained for the respective forms 1,2,3 and 4 from the table above as; 95.9701%, 96.4018%, 97.7203% and 99.6899% respectively.

Comparing the transition rates of girls in mixed schools to those in pure girls' schools in Kisumu west sub- County, it's realized that after one year the next state rates are 90.5229%<98.2089%, 82.4916%<96.7017%, 86.543%<97.7203%. The completion rates at form four was obtained as 97.9592%<99.6899%. for forms 1, 2, 3, and 4 respectively.

After two years later, the transition rates for girls in mixed schools can be compared to those in pure girl's schools as; 80.0654%<96.2686%, 80.8081%<96.4018%, 86.6427%<97.7203% and finally 97.9592%<99.6899% respectively. The rate of transition and completion of girls in pure public girl's school is greater than of girls mixed public secondary school.

In three years time, the transition absorption Markov chain was obtained as follows girls in mixed to girls in pure girls schools respectively; 78.4314%<95.9701%, 80.8081%<96.4018%, 86.6427%<97.7203%, 97.9592%<99.6899% Transition rate for girls in pure girls' school is higher than transition rates of girls in mixed secondary schools in the sub county Kisumu West, Kenya.

The study showed that on the school category, transition rates decreased with increasing level of schooling. However, transition rates were higher in pure public girls secondary schools compared to girls in mixed secondary schools in Kisumu West sub County of Kisumu County. The t-test result showed that there was a significance difference in the transition rates, $t = -0.726, p = 0.47 > 0.05$. The null hypothesis (H_0) rejected at 5% level of significance.

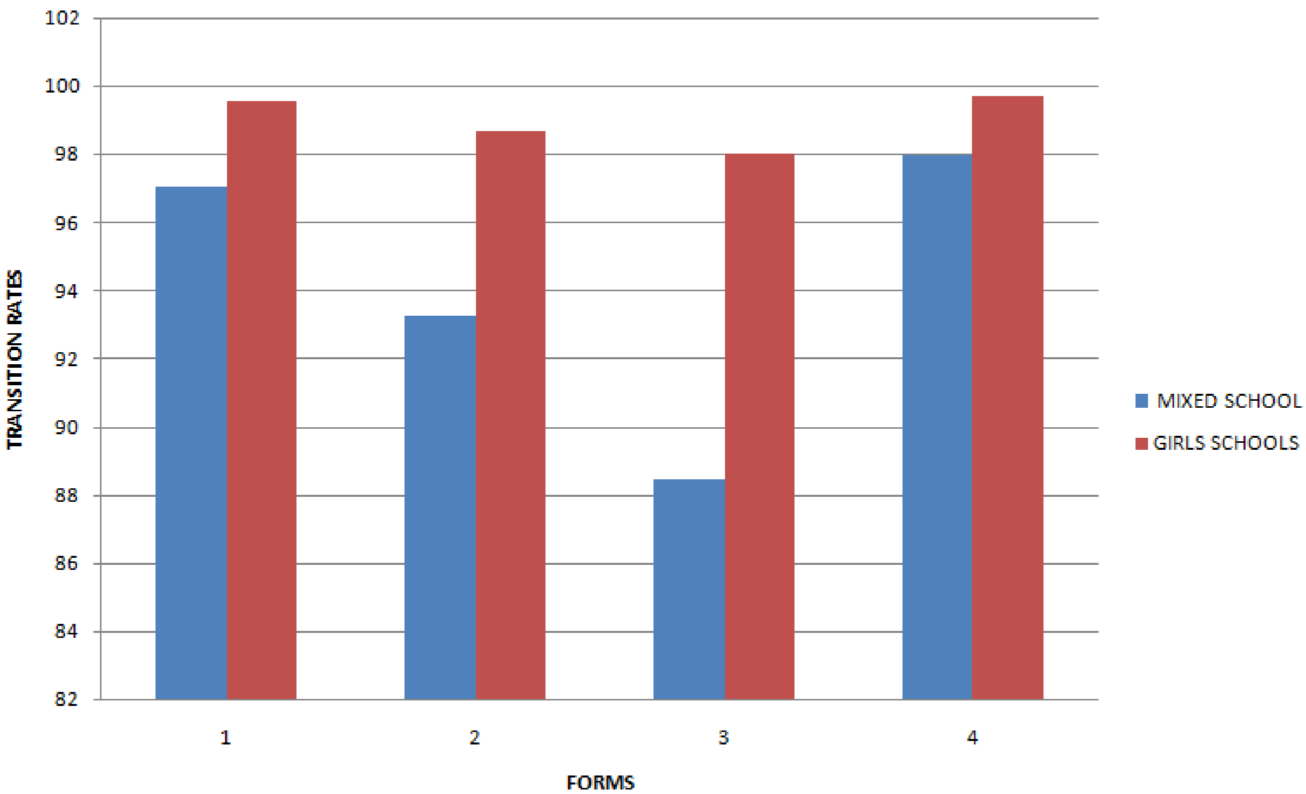

Figure 4.1: Graphical comparison of transition rates of girls in pure to mixed

4.6 Expected duration of study

4.6.1 The expected duration of study for girls in mixed school

Here the fundamental matrix N will be considered. The matrix gives the number of cycles that a subject resides in transient states before absorption, given a specified starting state. The fundamental matrix N is given as;

$$N = I + Q + Q^2 + \cdots = (I - Q)^{-1}$$

To compute N for the sub county, the following matrix shall be considered. For girls in mixed school, it is obtained by; $(I - Q_{sm})$.

$$(I - Q_{sm}) = \begin{pmatrix} 1.000000 & -0.970588 & 0.000000 & 0.000000 \\ 0.000000 & 1.000000 & -0.932660 & 0.000000 \\ 0.000000 & 0.000000 & 1.000000 & -0.884477 \\ 0.000000 & 0.000000 & 0.000000 & 1.00000 \end{pmatrix}$$

Its inverse is;

$$(I - Q_{sm})^{-1} = \begin{pmatrix} 1.000000 & 0.970588 & 0.905229 & 0.800654 \\ 0.000000 & 1.000000 & 0.932660 & 0.824916 \\ 0.000000 & 0.000000 & 1.000000 & 0.884477 \\ 0.000000 & 0.000000 & 0.000000 & 1.00000 \end{pmatrix}$$

The expected duration of study according to Beck and Pauker, 1983 for girls in mixed school is given by; $(1 - Q_{sm})^{-1}(1111)^T$ which gives:

$$\begin{pmatrix} 1.000000 & 0.970588 & 0.905229 & 0.800654 \\ 0.000000 & 1.000000 & 0.932660 & 0.824916 \\ 0.000000 & 0.000000 & 1.000000 & 0.884477 \\ 0.000000 & 0.000000 & 0.000000 & 1.00000 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3.676471 \\ 2.757576 \\ 1.884477 \\ 1.00000 \end{pmatrix}$$

This result gives the total expected duration of study of girls in mixed schools till completion. From the result, the expected duration of study for girls in Forms 1, 2, 3 and 4 are; 3.676471, 2.757576, 1.884477 and 1.000000 years respectively.

4.6.2 The expected duration of study for girls in pure girls' secondary school

On contrary, the expected duration of study for girls' in pure girl's secondary school is obtained as; $(I - Q_{pg})^{-1}(I)$.

$$(I - Q_{pg}) = \begin{pmatrix} 1.000000 & -0.995522 & 0.000000 & 0.000000 \\ 0.000000 & 1.000000 & -0.986507 & 0.000000 \\ 0.000000 & 0.000000 & 1.000000 & -0.980243 \\ 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{pmatrix}$$

Its inverse is;

$$(I - Q_{pg})^{-1} = \begin{pmatrix} 1.000000 & 0.995522 & 0.982089 & 0.962686 \\ 0.000000 & 1.000000 & 0.986507 & 0.967017 \\ 0.000000 & 0.000000 & 1.000000 & 0.980243 \\ 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{pmatrix}$$

According to Beck and Pauker, 1983, the expected duration of study for girls in pure girls school is given by; $(1 - Q_{sp})^{-1}(1111)^T$;

$$\begin{pmatrix} 1.000000 & 0.995522 & 0.982089 & 0.962686 \\ 0.000000 & 1.000000 & 0.986507 & 0.967017 \\ 0.000000 & 0.000000 & 1.000000 & 0.980243 \\ 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3.940293 \\ 2.953524 \\ 1.980243 \\ 1.00000 \end{pmatrix}$$

The total expected duration in school till completion. From the result, the expected duration of study for a student in Forms 1, 2, 3 and 4 are 3.940293, 2.953524, 1.980243 and 1.000000 years respectively Expected duration of study tells the exact period a student in a given form takes to graduation thus necessary for planning. There was a significance difference in duration of study of the school types (pure and mixed).

4.7 Absorbing Rates

Assuming that students will remain in the system indefinitely, then the absorbing rate is given by;

$$r_{i1}^{(\infty)} = \sum_{n=1}^{\infty} r_{i1}^{(n)} = (I + Q + Q^2 + \cdots)R = (I + Q)^{-1}R$$

The solution to this gives the absorbing rate under double absorbing states system. (Uche, 1980 and Musiga et al., 2011)

4.8 Absorbing rates by school type

4.8.1 Girls in mixed secondary

The absorbing rate was computed with respect to type of school. The absorbing rate under double absorbing states is given by $(I - Q)^{-1}R$.

The absorption rate for girls in mixed schools is $(I - Q_{sm})^{-1}R_{sm}$ which gives;

$$\begin{pmatrix} 1.000000 & 0.970588 & 0.905229 & 0.800654 \\ 0.000000 & 1.000000 & 0.932660 & 0.824916 \\ 0.000000 & 0.000000 & 1.000000 & 0.884477 \\ 0.000000 & 0.000000 & 0.000000 & 1.00000 \end{pmatrix} \begin{pmatrix} 0.000000 & 0.029412 \\ 0.000000 & 0.067340 \\ 0.000000 & 0.115523 \\ 0.979592 & 0.020408 \end{pmatrix} = \begin{pmatrix} 0.784314 & 0.215686 \\ 0.808081 & 0.191919 \\ 0.866427 & 0.133573 \\ 0.979592 & 0.020408 \end{pmatrix}$$

4.8.2 Girls in Pure secondary Secondary

For girls in pure girls school the double absorbing state is $(I - Q_{pg})^{-1}R_{pg}$;

$$\begin{pmatrix} 1.000000 & 0.995522 & 0.982089 & 0.962686 \\ 0.000000 & 1.000000 & 0.986507 & 0.967017 \\ 0.000000 & 0.000000 & 1.000000 & 0.980243 \\ 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{pmatrix} \begin{pmatrix} 0.000000 & 0.004477 \\ 0.000000 & 0.013493 \\ 0.000000 & 0.019757 \\ 0.996899 & 0.003101 \end{pmatrix} = \begin{pmatrix} 0.959701 & 0.040299 \\ 0.964018 & 0.035982 \\ 0.977203 & 0.022797 \\ 0.996899 & 0.003101 \end{pmatrix}$$

The result established that in the long run, absorption rates of girls in mixed secondary schools in Forms 1, 2, 3 and 4 in the years 2012 to 2015, were; 21.5686%, 19.1919%, 13.3573% and 2.0408% respectively dropped out of the system without attaining maximum qualification while for the girl students in pure public girls' secondary schools in the same order, 4.0299%, 3.5982%, 2.2797% and 0.3101% respectively dropped out. Also comparing transition rates, the same students in that order had 78.4314%, 80.8081%, 86.6427% and 97.9592% respectively for girls in mixed schools to successfully transit to the next state in the system while 95.9701%, 96.4018%, 97.7203% and 99.6899% for girls in pure public girls' schools respectively to successfully transit to the next state in the system.

This is helpful in avoiding certain states that may look permanent or impossible leaving for an effective transition.

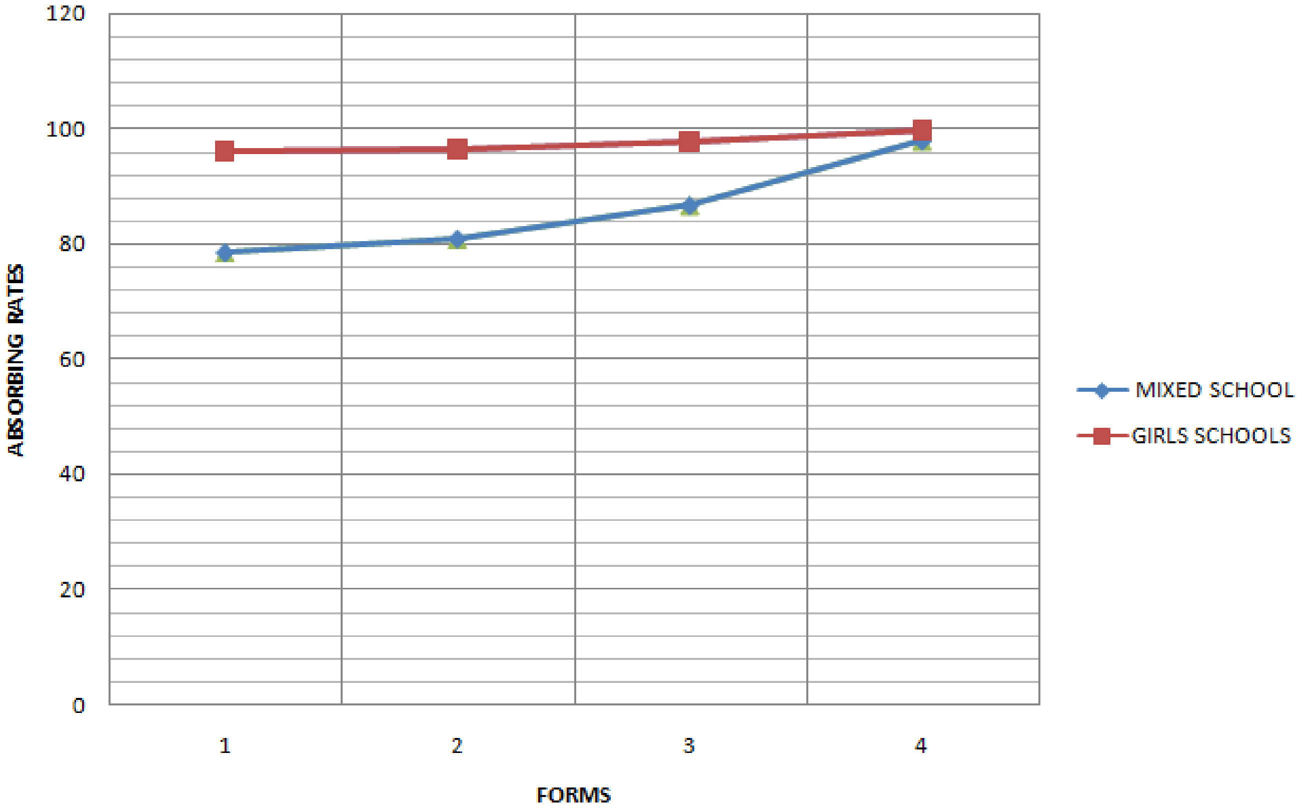

Figure 4.2: Absorbing rates of girls in pure girls vs mixed

Chapter 5

CONCLUSION AND RECOMMENDATION

5.1 Research findings

On the assumptions that there are no class repetitions in a closed hierarchical population, a Markov chain model has been used to study and analyze transition rates of girls in both pure public girls' schools and mixed secondary schools. Future projections have been obtained assuming time homogeneity.

From the findings, it was established that transition rates from form 1 to form 2 is higher in pure girls schools to girls in mixed schools with 0.995522:0.970588 respectively. Form three girls have the lowest transition rates to form four in both cases. It was lower in mixed schools as compared to pure girls' schools with rates of 0.884477: 0.980243 respectively.

Transition and completion rates in form four were the highest in both cases compared with 0.979592:0.996899 of girls in mixed to girls in pure girls' schools respectively.

Dropouts are relatively higher between form two and three in both cases, but higher in mixed schools compared to pure girls' schools.

5.2 Conclusion

In the single absorbing state case, (see Musiga et al 2010), all cadres of students were lumped together so it was not possible to determine the proportion of students who successfully graduated from the system. In this double absorbing states case, students who dropped out of the system before attaining the maximum qualification due to various reasons were distinctly separated from those who successfully graduated

The following conclusions were drawn from the above analysis and discussions;

- The transition rates for girls in pure public girls school is higher than that of girls in public mixed secondary schools in Kisumu West Sub county;
- The expected duration of study for girls in pure public girls' secondary schools is higher compared to ones in mixed public schools;
- The dropout rates for girls in mixed schools is higher than that of girls' in pure girls schools in Kisumu West Sub county;
- The transition rate of girls in mixed public schools from Form 2 to Form 3 is lower than that those in pure public girls schools

- The model can be used to project enrolment in the sub county though in a short duration;
- In the long run, completion and transition rates were found to be the same as the absorbing rates when repetition is not allowed.

The study showed that on the School Type category, transition rates decreased with increasing levels of schooling for both school categories. However, transition rates were higher in the pure public girls secondary schools compared to Mixed secondary schools and this led to the conclusion that the latter had put in place more measures to favour girl student transition compared to the former. The *t*-test results showed that there was statistically significant difference in the transition rates for Mixed and pure girls secondary schools.($t = -0.726, p = 0.470 > 0.05$). The null hypothesis ($H_0$) was rejected at 5% significance level.

5.3 Recommendations

From the above conclusions, the following recommendations were made;

1. There is an efficient transition rates of girls in pure public girls schools compared to mixed schools in Kisumu Sub County. Pure girls schools are therefore recommended for effective girl education.
2. Parents, BOM and sponsors should take their girls to pure public girls' schools and more attention given to them in form 2 and 3, more research should be carried out

for the cause of lowest transition.

3. Girls who have been taken to mixed schools should be closely monitored between forms 2 and 3 where the transition is relatively low. This study recommends that there is need to improve the conditions of learning in the single school types so that repetition rates are minimized.

4. Repetition and transfer of students should be abolished in secondary schools for effective transition.

5. Sponsorship be offered to needy girls in public pure girls schools in cases where parents cannot afford.

5.4 Further Research

1. Research should be done to get into the root cause of low transition of girls in mixed schools compared to pure public girls' schools between forms 2 and 3.

2. Considering the retention rates, research should be done to get the insight of the reasons as to why there is a lower transition rate in form 2.

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APPENDICES

Table 5.1: List of Public Secondary Schools in kisumu West Sub County

S/NO	SCHOOL	SCHOOL TYPE
1	AIC Olago Aluoch Alara Girls	PUBLIC GIRLS DAY
2	Bar Andingo	PUBLIC MIXED DAY
3	Bar Korumba Secondary	PUBLIC MIXED DAY
4	Bar Union Sec	PUBLIC MIXED DAY
5	Bishop Okoth Ojolla Girls	PUBLIC GIRLS BOARDING
6	Chulaimbo Sec	PUBLIC BOYS BOARDING
7	Dago Thim Mixed	PUBLIC MIXED DAY
8	Eluhobe	PUBLIC MIXED DAY
9	Gombe Kokulo	PUBLIC MIXED DAY
10	huma girls	PUBLIC GIRLS BOARDING
11	Kanyamedha Sec	PUBLIC MIXED DAY
12	Kawino Mixed	PUBLIC MIXED DAY
13	Kirembe Mixed	PUBLIC MIXED DAY
14	Kisian Secondary	PUBLIC MIXED DAY
15	Kuoyo Mixed	PUBLIC MIXED DAY

16	Lwala Kadawa	PUBLIC MIXED DAY
17	Maliera Sec	PUBLIC MIXED DAY
18	Mbaka Oromo	PUBLIC MIXED DAY
19	Maseno School	PUBLIC MIXED DAY
20	Obede Sec	PUBLIC MIXED DAY
21	Ogada Sec	PUBLIC MIXED DAY
22	Ogal Mixed	PUBLIC MIXED DAY
23	Oluowa	PUBLIC MIXED DAY
24	Ongalo Sec	PUBLIC MIXED DAY
25	Osiri Mixed	PUBLIC MIXED DAY
26	Sabembe Secondary	PUBLIC MIXED DAY
27	Sianda Mixed	PUBLIC MIXED DAY
28	Sinyolo Girls	PUBLIC GIRLS BOARDING
29	St Antony Dago Kokore	PUBLIC MIXED DAY
30	St Marks Obambo Sec	PUBLIC MIXED DAY
31	Tiengre Sec	PUBLIC MIXED DAY
32	Ulalo	PUBLIC MIXED DAY
33	Usare Mixed	PUBLIC MIXED DAY
34	Wachara Mixed	PUBLIC MIXED DAY
35	St. Gabriel Missionary	PRIVATE MIXED DAY
36	Jans Academy	PRIVATE MIXED DAY

Table 5.2: 2012 Secondary school enrolment in Kisumu West Sub-county

S/NO	SCHOOLS	SCHOOL TYPE	FORM 1		FORM 2		FORM 3		FORM 4		TOTAL
			B	G	B	G	B	G	B	G	
1	ALARA GIRLS	PUBLIC GIRLS DAY	-	66	-	60	-	44	-	40	210
2	BAR ANDINGO	PUBLIC MIXED DAY	8	20	13	20	18	19	13	12	123
3	BAR KORUMBA SEC.	PUBLIC MIXED DAY	20	32	16	25	15	10	-	-	118
4	BAR UNION SEC.	PUBLIC MIXED DAY	60	49	64	50	76	19	40	20	378
5	BISHOP OKOTH OJOLLA	PUBLIC GIRLS BOARDING	-	147	-	140	-	114	-	88	489
6	CHULAIMBO SEC.	PUBLIC BOYS BOARDING	210	-	185	-	185	-	160	-	740
7	DAGO THIM MIXED	PUBLIC MIXED DAY	22	33	29	27	18	22	20	14	185
8	ELUHOBE	PUBLIC MIXED DAY	29	30	26	48	32	33	38	34	270
9	GOMBE KOKULO	PUBLIC MIXED DAY	17	19	10	11	14	17	-	-	88
10	HUMA GIRLS	PUBLIC GIRLS BOARDING	0	138	-	136	-	100	-	106	480
11	KANYAMEDHA SEC	PUBLIC MIXED DAY	69	66	61	64	56	41	31	42	430
12	KAWINO MIXED	PUBLIC MIXED DAY	30	17	21	28	23	26	16	20	181
13	KIREMBE MIXED	PUBLIC MIXED DAY	22	28	25	33	20	20	22	20	190
14	KISIAN SEC.	PUBLIC MIXED DAY	26	28	35	31	34	40	30	31	255
15	KUOYO MIXED	PUBLIC MIXED DAY	77	68	80	50	55	53	72	45	500
16	LWALA KADAWA	PUBLIC MIXED DAY	30	22	39	27	37	13	12	10	190
17	MALIERA SEC	PUBLIC MIXED DAY	24	25	24	26	24	26	28	5	182
18	MBAKA OROMO	PUBLIC MIXED DAY	17	23	18	22	12	24	8	16	140
19	MASENO SCHOOL	PUBLIC MIXED DAY	367		384	-	310	-	347	-	1408
20	OBEDE SEC	PUBLIC MIXED DAY	42	35	26	19	23	19	15	16	195
21	OGADA SEC	PUBLIC MIXED DAY	33	30	53	20	25	15	39	20	235
22	OGAL MIXED	PUBLIC MIXED DAY	16	24	9	11	11	14	-	-	85
23	OLUOWA	PUBLIC MIXED DAY	5	32	30	30	29	32	33	28	219
24	ONGALO SEC	PUBLIC MIXED DAY	45	45	40	31	38	18	34	35	286
25	OSIRI MIXED	PUBLIC MIXED DAY	20	30	15	13	14	7	25	6	130
26	SABEMBE SEC.	PUBLIC MIXED DAY	10	17	14	12	16	13	-	-	82

S/NO	SCHOOLS	SCHOOL TYPE	FORM 1		FORM 2		FORM 3		FORM 4		TOTAL
			B	G	B	G	B	G	B	G	
27	SIANDA MIXED	PUBLIC MIXED DAY	40	60	25	31	14	11	23	26	230
28	SINYOLO GIRLS	PUBLIC GIRLS BOARDING	0	213	-	233	-	192	-	184	822
29	ST ANTONY DAGO KOKORE	PUBLIC MIXED DAY	80	58	60	48	57	50	51	46	450
30	ST MARKS OBAMBO SEC	PUBLIC MIXED DAY	76	66	78	70	77	69	74	43	553
31	TIENGRE SEC	PUBLIC MIXED DAY	56	61	24	50	11	31	26	18	277
32	ULALO	PUBLIC MIXED DAY	17	18	14	16	17	11	17	3	113
33	USARE MIXED	PUBLIC MIXED DAY	33	30	25	24	27	26	27	9	201
34	WACHARA MIXED	PUBLIC MIXED DAY	21	10	31	10	27	10	46	10	165
35	ST. GABRIEL MISSIONARY	PRIVATE MIXED DAY	21	18	14	23	24	23	14	15	152
36	JANS ACADEMY	PRIVATE MIXED DAY	20	11	18	6	16	16	5	15	107
	TOTAL		1563	1569	1506	1445	1355	1178	1266	977	9296