ANALYSIS OF YOUNG PEOPLE'S CAREER BY USE OF MARKOV CHAINS: CASE STUDY OF KISUMU CITY

BY

AOYI DANIEL OTIENO.

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DECLARATION

This research project is my own work and has not been presented elsewhere for a degree award in any other institution.

AOYI DANIEL OTIENO

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This project report has been submitted for examination with my approval as the university supervisor

Dr. EDGAR OUKO OTUMBA

Department of Statistics and Actuarial Science,

Maseno University

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God bless you in deed.

DEDICATION

I dedicate this work to GOD, my wife my daughters Mercy and Blessed.

ABSTRACT

It is the desire of every person to have a career. Not much thought has been taken by young people on how they can arrive at their future careers. The reason is mainly because they are not aware. As much as many people have attained their careers through education, not much consideration has been given to the other factors within education that leads one to his or her career. The study traced one's career from primary to present position. There are stages one follows to reach the career, which are called states in this study. The predicament of these young people is dealt with by use of Markov chains. Markov chains is a process that consists of finite number of states, which are four in this study. The four states that were considered in this study are, KCPE, KCSE, College and Career. KCPE, KCSE and College are transient states, while career is the final state. Regardless of where they started from, they ended up in Career with different proportions. Transitional probabilities were used to form transitional probability matrix. The matrix so formed was used to find Fundamental Matrix. The fundamental matrix has given the expected number of times the process was in each transient state, that is, the means. About 16 percent of those who did KCPE, 10 percent of those who did KCSE and 99 percent of college graduates got career. This shows that not many get into career after KCPE. Further still fewer young people get into career after KCSE, this may be because most of them prefer to proceed to college before career. The variances associated with transition among KCPE, KCSE and College are 0.821685, 0.037049 and 0.0069. Their respective standard deviations are 0.906468, 0.192481 and 0.083066. The low values indicate that the values did not deviate much from the expected, except for KCPE. Those who don't intend to go to college should rather identify a career path after KCPE.

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NOTATION

K.C.P.E Kenya Certificate of Primary Education. 4, 16, 20, 25, 28

K.C.S.E Kenya Certificate of Secondary Education. 4, 16, 18, 20, 25, 28

Chapter 1

INTRODUCTION

1.1 BACKGROUND

Career is defined by the Oxford English Dictionary as a person's "course or progress through life (or a distinct portion of life)". Career is what someone does for a living. It is a chosen pursuit; a profession or an occupation and people take different paths to arrive at their careers as well as ending up at different levels of education.

Career is an integral part of one's life and his/her life depends so much on his/her career. The desire and dream of every person is career and a lot of emphasis has been put on education as the way to having a good career all over the world.

Holyfield [10] said that education is critical to breaking the cycle of poverty, an opportunity of poor parents to obtain education for their children so that they can help them in old age . Education is seen as a way of career and hence more income to the family. When Kenya got independence in 1963, the first president Mzee Jomo Kenyatta desired to eradicate poverty, ignorance and illiteracy. He farther stated that education is fundamental in improving a country's general welfare.

Generally, there are three main stages or levels of education, that is primary, secondary and college.

People get into their careers at different academic levels. Not all children who enroll in class one go through all the three stages before they move to career. There are some that get into career even before completing primary education, others after completing primary education, others after completing secondary education and others after post secondary training.

Apart from primary graduates who go to career directly, there are some who go to college before career, some proceed to secondary, college before they settle on career and while the rest just drop out of the system. Also, apart from secondary graduates who go to career directly, there are those who go to college before career while the rest drop out of the system. There are drop outs in every stage. Some drop out by choice, others is because of circumstances. Some of the causes of drop out are sickness, death, poverty, cultural practices, peer pressure and academic weaknesses.

The level of education has a bearing on one's career. Higher level of education are associated with many benefits [6]. In contrast, more education is not always better. Therefore, as one pursues education, it is very important for one to think of the nature of career he/she wants to get into at an early stage of learning, like primary.

Most young people just go to school without serious thought of their destiny. They just choose subjects; maybe because they are offered in the school or their friends are doing the same or perhaps their parents/guardians want them to do them. It is necessary for young people to know the path to take to reach their careers.

Lack of serious thought about future career is a global problem. For instance in Southern California's San Fernado Valley School, they required every first-year student to submit a description of their plans for some post-secondary education or training [4]. The reports would be reviewed and counselors would talk to students with low expectations. Their good work is appreciated and welcomed, but there are things that counselors cannot do.

Traditionally, good education means good career, but presently, this is not the case. Statistics show that one in three college/university graduates could not find employment requiring a college/university degree. In fact, presently most jobs/careers advertised there is a lot of emphasis on experience. Experience rather than education seems to carry weight [4]. Also it is unfortunate that these days more people get jobs because of who they know and not what they know [18]. Mr. Mwai Kibaki, the former president of Kenya, in his speech during the 3rd global Youth Employment Summit 2006 at Kenyatta International Conference Centre, was concerned about unemployment among young people all over the world, especially developing countries like Kenya. He noted that this group of young people that make 75 percent of the total population should not be ignored.

In this study we want to determine the chances of a young person getting a career after achieving some level of education. At any one time a young person is in either of the four states. Primary, secondary, college state and finally a career.

We will also assume that a given observation period, say the t^{th} period , the probability that a young person is in a particular level of education depends only on his/her status at the $(t-1)^{th}$ period. Such a system is called a markov chain or markov process.

A Markov Chains model is used to analyze one's career given his/her level of education. A questionnaire was administered to people from different estates in Kisumu city. The data collected was analyzed to come up with transitional probabilities that forms a transitional probability matrix. The matrices formed were used to analyze the young people's career and establish probability of acquiring a career after every level of education.

1.2 STATEMENT OF THE PROBLEM

There are some people who get to careers at low academic levels while others get them at higher academic levels. Many young people start schooling without careers in mind. Most of them do not explore their future career until a later date of their schooling. Sometimes even parents and guardians also take their children to school without clear thought to future career. If people do not identify their careers early in life, they waste time and resources doing irrelevant things. That is, doing what they never aimed at. On the other hand, when they identify their career earlier, they are able to focus and develop their career paths early enough right from lower primary and hence reach their goal. Parents, governments and all who facilitate the education of the young people should know if their investment is in line with what these young people would like to be.

The current unemployment make it quite difficult to convince young people that there are gains in pursuing education to higher levels. The current study uses a markov chain model to analyze the chances of a young person getting career. The study to show the relationship between level of education and chances of getting a career.

1.3 OBJECTIVES OF THE STUDY

i. To determine the long-run prediction of the proportion of young people who get careers in each level of education..

ii. To use Markov chain model to analyze career choice of young people.

1.4 SIGNIFICANCE OF THE STUDY

The results of the study will enable the stakeholders, parents/guardians, statisticians/planners and teachers know how level of education affects career. This may help them know and plan how to advice young people. The study may encourage young people to explore their future career as early as primary school level.

1.4.1 ASSUMPTION

Young people take 8 years to graduate with Kenya Certificate of Primary Education (K.C.P.E), 4 years to graduate with Kenya Certificate of Secondary Education (K.C.S.E) and 4 years to train.

1.5 BASIC CONCEPTS

1.5.1 STOCHASTIC PROCESS

A stochastic process is a set of random variables $X_1, X_2, X_3, \ldots$ observed over time, where the subscript usually represents the time at which the random variables was observed. It is a process for which we do not know the outcome but can make estimates based on the probabilities of different events occurring over time.

1.5.2 MARKOV CHAINS

A Markov chain is a type of stochastic process where the future distribution of the given system depends only on the current value.

Given the state space $S = 1,2,3,\ldots,N$, a Markov chain is defined by a sequence of random variables, $X_i \in S$, for $i = 1,2,\ldots$ Such that:

$$Prob(X_{n+1} = x/X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n) = Prob(X_{n+1} = x_{n+1}/X_n = x_n) \quad (1.1)$$

The possible values of X_i form a countable set $S(S_1, \ldots, S_N)$ called state space of the chain. A state is the location of a particular people in the system at a particular time.

The property gives rise to a $N \times N$ transition matrix;

$$\mathbf{P} = P_{ij} = Prob(X_{n+1} = j/X_n = i) \quad (1.2)$$

It is a probability model describing the on going process of movement within the markov variable.

The Markovian stochastic process is memory-less. The knowledge of the current state is sufficient to predict what the future state will be and is independent of where the process has been in the past.

The Markov process can be classified according to characteristics of the state space being measured. A discrete or finite space is assumed for most purposes and implies there is a finite number of states that will be reached by the process.

The Markov process is also classified according to the time intervals of the observation of the process. The process may be observed at a restricted or discrete interval or can be observed continuously.

Markov chains are used to describe a process observed at discrete intervals with Markov process describes a process observed continuously. However Markov chains are special case of continuous time Markov processes where the process is continuous time Markov process observed at discrete intervals. therefore, Markov process can be used to collectively describe all processes and chains.

1.5.3 TRANSITION PROBABILITY

A transition probability is a probability of going from one state to another in one step, that is, P_{ij} = Probability of going from state i to state j in one step.

$$P(X_{t+1} = S_j : X = S_j) \tag{1.3}$$

for a markov chain with n states $S_1, \dots, S_n$.

$$P_{ij} \geq 0, \sum_{j=1}^{n} P_{ij} = 1 \tag{1.4}$$

1.5.4 MARKOV TRANSITION PROBABILITY MATRIX

Markov transition probability matrix is a square matrix describing the probabilities of moving from one state to another in a dynamic system. In each row are probabilities of moving from from the state represented by that row, to the other states. Thus the rows of a markov transition probability matrix each add to one.

It is formed from the records of moving from one state to another. It has the following properties:

(i) P_{ij} are conditional.

(ii) $P_{ij} \geq 0, \forall i, j$

(iii) $\sum_{k \in s} P_{ik} = 1$. If $\sum_{i \in s} P_{ik} = 1$, then **P** is a double stochastic.

(iv) For $X_0 = i$, the distribution of X_n is given by $P_{ij}^{(n)} = P(X_n = j/X_0 = i) = P(X_{n+m} = j/X_m = i)$

1.5.5 MARKOVIAN PROPERTY

A discrete time and discrete state space stochastic process is a markovian if and only if the conditional probabilities

$$Prob(X_{n+1}/X_0, \ldots, X_n) = \frac{Prob(X_0, \ldots, X_n, X_{n+1})}{Prob(X_0, \ldots, X_n)} \tag{1.5}$$

do not depend on $X_0, \ldots, X_n$ in full, but only on the most recent state X_n :

$$Prob(X_{n+1}/X_0, \ldots, X_n) = Prob(X_{n+1}/X_n)$$

The likelihood of going to any next state at time $n+1$ depends only on the state we find ourselves in at time n. The system is said to be memoryless.

For a Markovian chain one has

$$Prob(X_0, \ldots, X_T) = Prob(X_0) \prod_{n=1}^{T} Prob(X_n/X_{n-1}) \tag{1.6}$$

1.5.6 TRANSIENT STATE

Transient state is a transitional state. It is possible to move from it.

1.5.7 ABSORBING STATE

Absorbing state i is an absorbing if once entered cannot be left. That is,

$p_{ii} = 1$ and $p_{ij} = 0$ for $i \neq j$

1.5.8 ABSORBING MARKOV CHAIN

When the chain has at least one absorbing state and it is possible to transition from each non-absorbing state to some absorbing state (perhaps in multiple steps) then it is absorbing.

Properties of Absorbing Markov chain are:

(a) Regardless of the initial state, in a finite number of steps, the chain will enter its absorbing state and then stay in that state.

(b) The powers of transition matrix get closer and closer to some particular matrix.

(c) The long-run trend depends on the initial state.

(d) Let **P** be the transition matrix of an absorbing markov chain. The matrix can be arranged in this form.

$$\mathbf{P} = \begin{bmatrix} \mathbf{Q} & \mathbf{R} \\ \mathbf{O} & \mathbf{I}_k \end{bmatrix} \tag{1.7}$$

where $\mathbf{I}_k$ is the identity matrix where k equal the number of absorbing states. $\mathbf{O}$ is a matrix of all zeros.

1.5.9 CANONICAL FORM OF AN ABSORBING MARKOV CHAIN

Assume an absorbing Markov Chain has t transient states and r absorbing states, the transitional probability matrix **P** will take the following canonical form:

$$\mathbf{P} = \begin{bmatrix} \mathbf{Q} & \mathbf{R} \\ \mathbf{O} & \mathbf{I} \end{bmatrix} \tag{1.8}$$

Where

- $\mathbf{Q}$ is a $t \times t$ matrix. q_{ij} is the probability that a young person who is in state i at time $t-1$ will be in state j at time t.

- $i, j = 1, 2, 3, \ldots, t$

- $\mathbf{R}$ is a non-zero $t \times r$ matrix. r_{ik} is the probability that a young person in state i at time $t-1$ will get career with final state k at time t.

- $i = 1, 2, 3, \ldots, t$ and $k = 1, 2, 3, \ldots, r$

- $\mathbf{O}$ is an $r \times t$ zero matrix and

- $\mathbf{I}$ is and $r \times r$ identity matrix

$$\mathbf{P}^n = \begin{bmatrix} \mathbf{Q}^n & \mathbf{R}^n \\ \mathbf{O} & \mathbf{I} \end{bmatrix} \tag{1.9}$$

Where,

- $\mathbf{Q}^n$ is a $t \times t$ matrix is the probability that a young person who is in state i will be in state j, n years later. $i, j = 1, 2, \ldots, t$.

  Every entry of Q^n must approach zero as n approaches infinity.

- $\mathbf{R}^n$ stands for the $t \times r$ matrix in the upper right-hand corner of $\mathbf{P}^n$.

- $\mathbf{R}^n$=$(\mathbf{I} + \mathbf{Q} + \mathbf{Q}^2 + \ldots + \mathbf{Q}^{n-1})\mathbf{R}$ is a $t \times r$ matrix which gives the probability that a young person who is in state i will have a career k within n years. $i = 1, 2, 3, \ldots$ and $k = 1, 2, \ldots, r$.

- $\mathbf{O}$ is an $r \times t$ matrix of zeros which gives transition probabilities from absorbing states to non-absorbing states in n steps.

- $\mathbf{I}$ is $r \times r$ identity matrix which gives the transition probabilities between absorbing states in n steps.

1.5.10 THE FUNDAMENTAL MATRIX

The Fundamental matrix is defined as

$$\mathbf{F} = (\mathbf{I} - \mathbf{Q})^{-1} = \mathbf{I} + \mathbf{Q} + \mathbf{Q}^2 + \ldots \tag{1.10}$$

where

- $\mathbf{I}$ has the same size as $\mathbf{Q}$
- The ij entries in $\mathbf{F}$ shows the number of visits to state j that are expected to occur before absorption, given that the current state is i.

The product **FR** gives the matrix of probabilities that an initial non-absorbing state will lead to a particular absorbing state.

1.5.11 HADAMARD PRODUCT

Hadamard Product is a binary operation that takes two matrices of the same dimension, and produces another matrix where each element ij is the product of elements ij of the original two matrices.

Let $\mathbf{A}$ and $\mathbf{B}$ be $m \times n$ matrices. The Hadamard Product of $\mathbf{A}$ and $\mathbf{B}$ is defined by $[\mathbf{AB}]_{ij} = [\mathbf{A}]_{ij}[\mathbf{B}]_{ij}$ for all $1 \leq i \leq m, 1 \leq j \leq n$. For example,

If $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

The first t states are transient and the last r states are absorbing. The ij^{th} entry, p_{ij}^n of the matrix $\mathbf{P}^n$ gives the probability that the Markov chain, starting in state s_i will be in state s_j after n steps by Chapman-Kolmogorov theorem. The canonical form of the matrix $\mathbf{P}^n$ is given as:

and $\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$

then

$$\mathbf{AB} = \begin{bmatrix} a_{11}b_{11} & a_{12}b_{12} & a_{13}b_{13} \\ a_{21}b_{21} & a_{22}b_{22} & a_{23}b_{23} \\ a_{31}b_{31} & a_{32}b_{32} & a_{33}b_{33} \end{bmatrix} \tag{1.11}$$

1.5.12 VARIANCE AND STANDARD DEVIATION

The Variance is the average squared differences from the mean. **The Standard deviation** is a measure of how spread out a distribution is. It is the square root of Variance. They are measures of variability.

1.5.13 SAMPLE AND SAMPLING

Sample is a subset of your population by which you select to be participants in your study. Sampling is the selecting a portion of the population in your research area, which will be a representation of the whole population.

1.5.14 STRATIFIED RANDOM SAMPLING

It is a method of sampling that involves the division of a population into smaller groups known as strata.

1.5.15 MULTINOMIAL DISTRIBUTION

A Multinomial Distribution is the probability distribution of the outcomes from a multinomial experiment. A multinomial experiment is a statistical experiment that has the following properties.

(a) The experiment consists of a fixed number of trials.

(b) All outcomes of each trial must be classified into exactly one of several (k) categories.

(c) The trials are independent.

(d) The probabilities for the different categories remain constant for each trial.

Chapter 2

LITERATURE REVIEW

2.1 INTRODUCTION

Markov chains was named after the Russian mathematician Andrew Andreevich Markov $(1856-1922)$ who started the theory of stochastic processes. A Markov chain is a type of stochastic process where the future distribution of the given system depends only on the current value. Physical or mathematical systems that have n possible states and at any one time, the system is in one and only one of its n states motivate the study of stochastic processes. Further assuming that at a given observation period, say k^{th} period, the probability of the system being in a particular state depends only on its status at the $(k-1)^{th}$ period then a Markov Chain can be used to make future predictions. A number of studies were done.

2.2 REVIEW OF RELEVANT LITERATURE

Markov chains are applicable in many areas, for example, internet applications, economics and finance, gambling, music, baseball and statistical testing [8], population genetics [2], geology and stratigraphy [13]; Harbaugh and Bonham-Cater [7]; Norris, [15]; Smeach and Jernigan, [16]. Anthony and Taylor [1] also used Markov Chain to explore air pollution levels. Many researchers have used it in career policy issues. They include, Kim and Smith [12], Heyman and Sobel [9], Zanakis and Maret [19], Calantone and Darmon [5].

Markov chain has also become a standard tool of medical decision making.

Damjan, Vasja and Darko [17] did a study on manpower planning for the Slovenian armed forces. They identified 120 types of military segments. then administrative data was used to estimate the transitions between the segments for the 2001-2005 period. Markov chain models were then used and 5-year, 10-year and 20-year projections were calculated. significant gaps were found in the projected sizes of the segments compared to the official targets.

“What is it that influences children one way or another?”[3].

Igboanugo and Onifade in [11] used the Markovian statistical tool to unravel the dynamics of staff stock and flow in a typical first generation Nigerian University. They used a forty-year data of staff transition within the six well defined states which were condensed from sixteen states. The six states were transformed into frequency distribution which was ultimately used to estimate the transition probability matrix. The states are the status a Markov chain staff undergoes in the transition process towards the final position of retirement, for those who would go there.

Igboanugo and Onifade [11] obtained transitional probabilities which they organized in canonical form in order to conform to the theory of Markov Chain.

Igboanugo and Onifade [11] used a quota sampling method. Markov chain was used to analyse the data collected. They came up with Markov Transitional Diagram which was developed to a transitional probability matrix. Then they used the transitional probabilty matrix to derive the fundamental matrix. The fundamental matrix was used to get the various means and variances in transient states.

The findings were that 47 percent of newly recruited staff exit the employment system through the normal retirement while 53 percent leave by either voluntary withdrawal or wastage. Some of the causes of attrition is staff moving to greener pastures and some disciplinary cases.

Over the school's front door at Ridge School of Technical Arts is the saying, "Work is one of our greatest blessings. Everyone should have an honest occupation."[16].

Some young people may identify their career early enough, and get education towards the goal, but unfortunately miss it because by the time they graduate the career opportunities have reduced or all non-existent [4]. J. Rawe reported that mechanical engineering student Elisabeth Rareshide, aged 22,who graduated with an "A"average from Rice University in Houston had to scramble to get work [4], still confirms that education is still one the requirements in ones career.

Young people must honestly evaluate where their best opportunities lie and which ones they can use to their best advantage [4].

Employees need skill in order to attain career. Many organizations have developed career development programs to support and motivate employees to continuous and life long learning (Filpszak and Hequet cited in Lankard/Brown's article 1996).

Joann Deml, career advisor for the University of Wisconsin Stout and Lia Reich (2001), graduate student in counseling, discussed the factors that enrich young people's potential for career success. This will enable them identify the career with maximum satisfaction. Though young people will keep on moving from one job to the other, they need to settle where they can have optimum output. This is a practice that can continue the rest of their lives (*J.Deml and L.Reich*, *personal communication*, *June* 18, 2000)

There are several students both in schools and colleges/university who are still struggling with career choice [14]. This has led to some going into careers which they never desired. Hence change of career because of lack of job satisfaction.

It would be necessary for young people to identify people with careers they want to pursue and let them know the path they should take to reach where they want to be [14].

The analysis by Markov chains can help a young man to know the career path to take.

Chapter 3

RESEARCH METHODOLOGY

3.1 INTRODUCTION

The chapter defines the location of the study, target population, study sample, research design and model development. A Markov Chain is used with a 4×4 Transition matrix. A state is the level of education attained by the young person (K.C.P.E, K.C.S.E and College) and their eventual Career. The first three states are transient and the last one is an absorbing one. The four states enabled the researcher to get a model to study how people transit from a level of education to career.

3.2 LOCATION OF THE STUDY

The study was done in Kisumu city. This is the third largest city in Kenya, on the shores of Lake Victoria. It was formerly called Port Florence. It is at an elevation of 1131 metres. It is a cosmopolitan city with a lot of commercial activities.

3.3 TARGET POPULATION

Target population was people with career between 15 and 39 years in Kisumu city. It has a population of about 216479 people, according to World population review.

3.4 THE STUDY SAMPLE

According to census conducted by the Kenya National Bereau of standards in 2009, the population of Kisumu city was 390,164 people. Out of which 185,821 were people within the ages of 15 and 39 years. With the national population growth rate of 2.8 percent per year, it is projected that in 2014 (the year the study was done) the population in Kisumu city would be about 486,622 people and the population of young people would be 213,334 people.

Stratified sampling was used to divide the city into seven main zones or estates. That is, Milimani, Mamboleo, Manyatta, Nyalenda, Otonglo, Nyamasaria and City centre.

Simple random sampling was used to pick 410 young people with career and at least KCPE certificate from each of the seven estates. Hence a total of 2870 people were interviewed.

3.5 THE MODEL

3.5.1 INTRODUCTION

Markov chains is a process that consists of finite number of states and some known probabilities p_{ij}, where p_{ij} is the probability of moving from state i to j. Sometimes these states are called current state and next state. Movement from one state to another is called transition or step.

The Markov Chain Model used has t non-absorbing states $(1,2,\ldots,t)$ corresponding to education levels and r absorbing states corresponding to career.

So $N = t + r$, where N is the total possible states from KCPE to Career.

This study addresses the steps for reaching one's career. Markov chains is a tool to be used to describe states thereby predicting one's career prospect. The independence from the previous states meets the criteria for the Markov property. Therefore, this model is called markov process.

For this study, it is required that the random variable be discrete and have a finite number states, in this case four. Markov chains will be used to analyze the four states. The Markov property states that the conditional probability distribution for the system at the next step (and in fact at all future steps) depends only on the current state of the system, and not additionally on the state of the system at previous steps. This simply means that the probability of being in state i on the nth step given that we know the state for all previous steps $(1,2,3,\ldots,n)$ is the same as just knowing the state during step n (the most recent event). The career one will pursue will depend on his or her present state. It does not depend on the previous steps (previous careers or previous academic achievement).

3.5.2 TRANSITIONS

State j is reachable or accessible from state i, $(i \rightarrow j)$ if there is a path from i to j. In this study, these levels of education and stages are called states. That is, KCPE is state 1 (S_1), K.C.S.E is state 2 (S_2), College is state 3 (S_3) and Career is state 4 (S_4). One's career depends on his or her latest level of education. The changes of state in the system are called transitions, and the probabilities associated with various state-changes are called transition probabilities. The set of all states and transition probabilities completely characterizes a Markov chain.

3.5.3 MATRIX OF TRANSITION

P_{ij} is the probability that a young person in state i in time $t-1$ will be in state j at the time t.

Let $n_{ij}(t)$ represent the number of young people in state i at time $t-1$ who will be in state j in time t and $n_i(t-1)$ represent the number of young people in state i at time $t-1$.

Assuming the multinomial distribution, the transition probabilities were estimated by

$$P_{ij} = \frac{n_{ij}(t)}{n_i(t-1)} \tag{3.1}$$

where $i, j = 1, 2, \ldots, N$.

The fraction above is the proportion of young people who were in state i at time $t-1$ who ended up in state j at time t.

3.5.4 MATRIX OF TRANSITIONAL PROBABILITIES

From the transitional probabilities, a matrix of transitional probabilities was got. Transitional probability matrix $\mathbf{P} = [p_{ij}]$, i, j. Since p_{ij} are conditional probabilities, they must satisfy the properties for all i and j for all i.

The Transition matrix is a 4×4 matrix containing the transition probabilities p_{ij} for moving from the current state in the process to the next state is shown below.

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \\ p_{41} & p_{42} & p_{43} & p_{44} \end{bmatrix} \tag{3.2}$$

The matrix $\mathbf{P}$ is said to be stochastic because all p_{ij} values lies within the interval $(0,1)$, inclusive and the sum of each row adds identically to 1. The above transitional probability matrix to be written in canonical form:

$$\begin{bmatrix} \mathbf{Q} & \mathbf{R} \\ \mathbf{O} & \mathbf{I} \end{bmatrix}$$

Where

- $\mathbf{Q}$ is a 3×3 matrix
- $\mathbf{Q}$ is the square matrix with rows and columns representing the transient states of the system. It is the probability of transitioning from some transient state to another.

- q_{ij} is the probability that a young person who is in state i at time $t-1$ will be in state j at time t. where $i, j = 1, 2, 3$

- **R** is a non-zero 3×1 matrix.

Transitions from non-absorbing states to absorbing states are possible. Its rows represent the transient states and whose columns represent the absorbing state. The entries represent the probability of being absorbed from the transient state to the respective absorbing state in one transition. It is the probability of transitioning from some transient state to some absorbing state.

r_{ik} is the probability that a young person in state i at time $t-1$ will get career with final state k at time t. where $i = 1, 2, 3$ and $k = 1$. **R** is a long-run distribution to career.

O is a 1×3 zero matrix. Matrix of zeros represent transition from absorbing state to non-absorbing state which is impossible. The probability is zero.

Transitions between non-absorbing states are possible.

I is 1×1 identity matrix with the rows and columns being the absorbing state of the system. Identity matrix is used to represent transition probabilities between absorbing states. The probability is one.

There is one absorbing state which is career. The other three are transient states, which are K.C.P.E, K.C.S.E and College.

3.5.5 THE N-STEP TRANSITION MATRIX

The entry $p_{ij}^{(n)}$ of the matrix $\mathbf{P}^n$ is the probability of being in state s_j after n steps, when the chain is started in state s_i. The n-step transition probability matrix takes the form

$$\mathbf{P}^n = \begin{bmatrix} \mathbf{Q}^n & \mathbf{R}^n \\ \mathbf{O} & \mathbf{I} \end{bmatrix}$$

n-step transition probability matrix can be found by multiplying the single -step probability matrix by itself n times. The solution of n-step transition matrix gives the state of a young person n-step (years later). The elements of n-step transition probability matrix represents the probabilities that a young person in a given state will be in the next state n-steps later.

The form $\mathbf{P}^n$ shows that the entries of $\mathbf{Q}^n$ gives the probabilities for being in each of the transient states after n steps for each possible transient starting state.

$\mathbf{Q}^n$ is a 3×3 matrix is the probability that a young person who is in state i will be in state j, n steps later. $i, j = 1, 2, 3$. Every entry of $\mathbf{Q}^n$ must approach zero as n approaches infinity.

In an absorbing Markov chain, the probability that the process will be absorbed is 1. That is, $\mathbf{Q}^n \to 0$ as $n \to \infty$.

$\mathbf{R}^n$ stands for the 3×1 matrix in the upper right-hand corner of $\mathbf{P}^n$. $\mathbf{R}^n$=$(\mathbf{I}+\mathbf{Q}+\mathbf{Q}^2+\ldots+Q^{n-1})\mathbf{R}$ is a 3×1 matrix which gives the probability that a young person who is in state i will have a career k within n years. $i = 1, 2, 3$ and $k = 1$.

$\mathbf{O}$ is a 1×3 matrix of zeros which gives transition probabilities from career to non-absorbing states(KCPE, KCSE and College) in n steps.

$$\mathbf{P}^2 = \begin{bmatrix} \mathbf{Q}^2 & (\mathbf{I}+\mathbf{Q})\mathbf{R} \\ \mathbf{O} & \mathbf{I} \end{bmatrix}$$

$$\mathbf{P}^3 = \begin{bmatrix} \mathbf{Q}^3 & (\mathbf{I}+\mathbf{Q}+\mathbf{Q}^2)\mathbf{R} \\ \mathbf{O} & \mathbf{I} \end{bmatrix}$$

$$\mathbf{P}^4 = \begin{bmatrix} \mathbf{Q}^4 & (\mathbf{I}+\mathbf{Q}+\mathbf{Q}^2+\mathbf{Q}^3)\mathbf{R} \\ \mathbf{O} & \mathbf{I} \end{bmatrix}$$

$\mathbf{I}$ is 1×1 identity matrix which gives the transition probabilities between career in n steps.

In general,

$$\mathbf{P}^n = \begin{bmatrix} \mathbf{Q}^n & (\mathbf{I}+\mathbf{Q}+\mathbf{Q}^2+\ldots+\mathbf{Q}^{n-1})\mathbf{R} \\ \mathbf{O} & \mathbf{I} \end{bmatrix} \tag{3.3}$$

$p_{ij}^{(2)}$ represents the probability of transition a young person from state i to j in two steps. $p_{ij}^{(n)}$ represents the n-step transition probability of a young person from state i to state j for

$n = 2, 3, \ldots$

As the powers of **P** is higher and higher, a young person tends towards a career. The probability is 1 that the young person will eventually get a career.

In a finite number of steps the young person will enter into career and then stay in that state regardless of the original state of an absorbing Markov chain.

The powers of the transition probability matrix **P** get closer and closer to some particular matrix.

3.5.6 FUNDAMENTAL MATRIX

F is the **expected number of visits** to a transient state j starting from a transient state i, before being absorbed. **F** which is the mean, gives the expected number of times in transient states that a young person at KCPE, KCSE or College transit within one another before they eventually get a career after a long-run transition.

Since

$$(\mathbf{I}-\mathbf{Q})(\mathbf{I}+\mathbf{Q}+\mathbf{Q}^2+\ldots+\mathbf{Q}^{n-1}) = \mathbf{I}-\mathbf{Q}^n \tag{3.4}$$

Given that as n approaches infinity, $\mathbf{Q}^n$approaches zero.

Then

$$(\mathbf{I}+\mathbf{Q}+\mathbf{Q}^2+\ldots+\mathbf{Q}^{n-1}) = \frac{1}{\mathbf{I}-\mathbf{Q}} = (\mathbf{I}-\mathbf{Q})^{-1} \tag{3.5}$$

Therefore

$$\mathbf{P}^n = \begin{bmatrix} \mathbf{Q}^n & (\mathbf{I}-\mathbf{Q})^{-1}\mathbf{R} \\ \mathbf{O} & \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{O} & (\mathbf{I}-\mathbf{Q})^{-1}\mathbf{R} \\ \mathbf{O} & \mathbf{I} \end{bmatrix} \tag{3.6}$$

In an absorbing markov chain, the matrix $\mathbf{I}-\mathbf{Q}$ has an inverse **F** and $\mathbf{F} = (\mathbf{I}-\mathbf{Q})^{-1} = \mathbf{I}+\mathbf{Q}+\mathbf{Q}^2+\ldots$.

$\mathbf{F} = (\mathbf{I}_3-\mathbf{Q})^{-1}$, where $\mathbf{I}_3$ is a 3×3 identity matrix.

The ij-entry f_{ij} of the matrix **F** is the expected number of times a young person is in transient state s_j, given that he/she starts in transient state s_i. **F** is called the fundamental matrix for **P**.

VARIANCE ON THE NUMBER OF VISITS

Variance on the number of visits to a transient state j with starting at a transient state i before being absorbed is the ij-entry of the matrix,

$$\mathbf{Var}(f_{ij}) = \mathbf{F}(2\mathbf{F}_{dg} - \mathbf{I}) - \mathbf{F}_{sq} \tag{3.7}$$

I is an identity matrix. $\mathbf{F}_{dg}$ is the diagonal matrix of **F**. $\mathbf{F}_{sq}$ is square of each entry of **F**.

Given that **F** is the expectations,
$\mathbf{Var}(f_{ij})$ is variance of the movements of the young person within the KCPE, KCSE and College before career.

TIME OF ABSORPTION - EXPECTED NUMBER OF STEPS

t is the expected number of steps before being absorbed when starting in transient state i. Let t_i be the expected number of steps before a young person gets a career, given that he/she starts in state s_i, and let t be the column vector whose ith entry is t_i

Then

$$\mathbf{t} = \mathbf{Fc} \tag{3.8}$$

where **c** is a column vector all of whose entries are 1 and $\mathbf{t} = t_1, \ldots, t_k$ is a vector where t_i is the expected number of steps until young person gets a career starting from state i.

VARIANCE OF THE NUMBER OF STEPS

.

The variance on the number of steps a young person makes before being in career when starting in transient state i is the ith entry of the vector

$$(2\mathbf{F}-\mathbf{I})\mathbf{t}-\mathbf{t}_{sq} \tag{3.9}$$

where **F** is the fundamental matrix **I** is a unit matrix $\mathbf{t}_{sq}$ is the square of evry entry of t

The expected number of times in any of the transient states for a starting state s_i is the sum of all the entries in the ith row of **F**

Therefore,

$$\mathbf{t}_2 = (2\mathbf{F}-\mathbf{I})\mathbf{t}-\mathbf{t}_{sq}$$

$\mathbf{t}_2$ represents the variances associated with transition of young people among the three non-absorbing states.

CAREER

Fundamental matrix can be used to compute the ultimate probability of being absorbed into each of the absorbing states by the following formular:

$$\mathbf{B} = \mathbf{FR} = (1-\mathbf{Q})^{-1}\mathbf{R} \tag{3.10}$$

B is a 3×1 matrix with entries b_{ij}. b_{ij} is the probability that a young person will have career in state s_j if he/she starts in the transient state s_i.

Given that $\mathbf{Q}^n \to 0$ as $n \to \infty$, thus

$$\mathbf{P}^{\infty} = \begin{bmatrix} \mathbf{Q}^n & \mathbf{R}^n \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \tag{3.11}$$

B=FR gives the probabilities that if a young person was originally in KCPE, KCSE or College, he/she ends in a career. **B** provides the long-run distribution of career of young people.

Chapter 4

DATA ANALYSIS, PRESENTATION AND DISCUSSION

4.1 INTRODUCTION

A Markov Chain is used to analyze the data gathered from people in Kisumu city.

The 2870 people who were interviewed had career and at least a KCPE certificate. The Table 4.1 shows the graduates at different levels.

Table 4.1: Graduates at different levels

Level of education	Number of people
KCPE	2870
KCSE	2249
College	2057
People with Career	2739

Out of the 2870 who did K.C.P.E 2249 proceeded to do K.C.S.E, 107 went to college, 461 went direct for career and the rest 53 did not proceed. Out of the 2249 who did K.C.S.E 1950 proceeded to college, 235 went for career and the rest 64 did not proceed. And out of 2057 who graduated from college 2043 proceeded for career, while the rest 14 remained jobless. A total of 2739 got into career. At different levels young people dropped out due to various reasons. Some of the reasons are, terminal illnesses, indiscipline, poverty, being orphaned, relocation, incapacitation, degrading cultural practices (especially for girls), weaknesses in academics, social problems and other family problems. The reasons above have caused learners to drop out of learning institutions before achieving their dreams.

The frequency distribution table below shows the four states and the number of people in each state. It also shows the number that transist from one state to the next.

Table 4.2: States

Transitional states	$\mathbf{S}_1$	$\mathbf{S}_2$	$\mathbf{S}_3$	$\mathbf{S}_4$
$\mathbf{S}_1$	53	2249	107	461
$\mathbf{S}_2$	0	64	1950	235
$\mathbf{S}_3$	0	0	14	2043
$\mathbf{S}_4$	0	0	0	131

Transitional probability matrix $\mathbf{P} = [p_{ij}], 1 \leq i, j \leq 4$. Since p_{ij} are conditional probabilities, they must satisfy the properties

$$p_{ij} \geq 0$$

for i and j

Transition matrix is a 4×4 matrix containing the transition probabilities p_{ij} for moving from the current state in the process to the next state.

P_{ij} is the probability that a young person in state i in time $t-1$ will be in state j at the time t.

Let $n_{ij}(t)$ represent the number of young people in state i at time $t-1$ who will be in state j in time t and $n_i(t-1)$ represent the number of young people in state i at time $t-1$.

Assuming the multinomial distribution, the transition probabilities were estimated by

$$P_{ij} = \frac{n_{ij}(t)}{n_i(t-1)}$$

where $i, j = 1, 2, \ldots, 4$.

The initial transition probabilities were used to make transition matrix shown below:

$$\mathbf{P} = \begin{bmatrix} 53/2870 & 2249/2870 & 107/2870 & 461/2870 \\ 0 & 64/2249 & 1950/2249 & 235/2249 \\ 0 & 0 & 14/2057 & 2043/2057 \\ 0 & 0 & 0 & 2739/2739 \end{bmatrix}$$

$$= \begin{bmatrix} 0.018467 & 0.783624 & 0.037282 & 0.160627 \\ 0 & 0.028457 & 0.867052 & 0.104491 \\ 0 & 0 & 0.006806 & 0.993194 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.1}$$

The matrix **P** is said to be stochastic because all p_{ij} values lies within the interval $(0, 1)$, inclusive and the sum of each row adds identically to 1.

p_{ij} gives transitional probabilities from state i to state j. The above transitional probability matrix is written in canonical form:

$$\begin{bmatrix} \mathbf{Q} & \mathbf{R} \\ \mathbf{O} & \mathbf{I} \end{bmatrix}$$

Where

$$\mathbf{Q} = \begin{bmatrix} 0.018467 & 0.783624 & 0.037282 \\ 0 & 0.028457 & 0.867052 \\ 0 & 0 & 0.006806 \end{bmatrix} \tag{4.2}$$

Q is a 3×3 matrix, q_{ij} is the probability that a young person who is in state i at time $t-1$ will be in state j at time t. Where $i, j = 1, 2, 3$.

The probability that a young person with K.C.P.E will remain with K.C.P.E, will have K.C.S.E and will have college certificate is 0.018467, 0.783624 and 0.037282 respectively. The probability that a young person with K.C.S.E will remain with K.C.S.E and will have college certificate is 0.028457 and 0.867052 respectively. The probability that a young person with college certificate will remain with college certificate is 0.006806.

$$\mathbf{R} = \begin{bmatrix} 0.160627 \\ 0.104491 \\ 0.993194 \end{bmatrix} \tag{4.3}$$

R is a non-zero 3×1 matrix, r_{ik} is the probability that a young person in state i at time $t-1$ will get career with final state k at time t. Where $i = 1,2,3$ and $k = 1$. **R** is a long-run distribution to absorbing state. The probability that a young person with K.C.P.E, K.C.S.E and college certificate getting career is 0.160627, 0.104491 and 0.993194 respectively.

$$\mathbf{O} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \tag{4.4}$$

O is a 1×3 zero matrix and

$$\mathbf{I} = \begin{bmatrix} 1 \end{bmatrix} \tag{4.5}$$

I is 1×1 identity matrix. Probability that one with career will have career is 1. There is one absorbing state which is career. There are three transient states, which are KCPE, KCSE and College. The entry $p_{ij}^{(n)}$ of the matrix $\mathbf{P}^n$ is the probability of being in state s_j after n steps, when the chain is started in state s_i. The n-step transition probability matrix takes the form.

$$\mathbf{P}^n = \begin{bmatrix} \mathbf{Q}^n & \mathbf{R}^n \\ \mathbf{O} & \mathbf{I} \end{bmatrix}$$

$$\mathbf{P} = \begin{bmatrix} 0.018467 & 0.783624 & 0.037282 & 0.160627 \\ 0 & 0.028457 & 0.867052 & 0.104491 \\ 0 & 0 & 0.006806 & 0.993194 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{P}^2 = \begin{bmatrix} 0.00341 & 0.036771 & 0.680385 & 0.282503 \\ 0 & 0.008100 & 0.030575 & 0.968616 \\ 0 & 0 & 0.000046 & 0.999955 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{P}^3 = \begin{bmatrix} 0.000063 & 0.003718 & 0.036640 & 0.962647 \\ 0 & 0.000023 & 0.000910 & 0.999068 \\ 0 & 0 & 0.000000 & 1.000000 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{P}^4 = \begin{bmatrix} 0.000012 & 0.000155 & 0.003475 & 0.999418 \\ 0 & 0.0000000 & 0.000026 & 0.999975 \\ 0 & 0 & 0.000000 & 1.000000 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

As n increases the chances of young people getting career increases. That is, it becomes one.

4.2 MEANS AND VARIANCES

From **P**,

$$\mathbf{Q} = \begin{bmatrix} 0.018467 & 0.783624 & 0.037282 \\ 0 & 0.028457 & 0.867052 \\ 0 & 0 & 0.006806 \end{bmatrix}$$

Its fundamental matrix becomes,

$$\mathbf{F} = (\mathbf{I}_3 - \mathbf{Q})^{-1}$$

where $\mathbf{I}_3$ is a 3×3 identity matrix.

$$\begin{aligned}
\mathbf{I}_3 - \mathbf{Q} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.018467 & 0.783624 & 0.037282 \\ 0 & 0.028457 & 0.867052 \\ 0 & 0 & 0.006806 \end{bmatrix} \\
&= \begin{bmatrix} 0.981533 & -0.783624 & -0.037282 \\ 0 & 0.971543 & -0.867052 \\ 0 & 0 & 0.993194 \end{bmatrix} \\
\mathbf{F} &= \frac{1}{0.947111} \begin{bmatrix} 0.964931 & 0.778291 & 0.715664 \\ 0 & 0.974853 & 0.953602 \\ 0 & 0 & 0.953602 \end{bmatrix} \\
&= \begin{bmatrix} 1.018815 & 0.821752 & 0.755628 \\ 0 & 1.029291 & 1.006853 \\ 0 & 0 & 1.006853 \end{bmatrix} \qquad (4.6)
\end{aligned}$$

f_{ij} is the average number of times that a young person starting from s_i is in s_j. The average number of times that a young person who started from KCPE visit KCPE, KCSE and College is $1.018815 \approx 1$, $0.821752 \approx 1$ and $0.755628 \approx 1$ respectively.

The average number of times that a young person who started from KCSE visit KCPE, KCSE and College is 0, $1.029291 \approx 1$ and $1.006853 \approx 1$ respectively.

The average number of times that a young person who started from College qualification visit KCPE, KCSE and College is 0, 0 and $1.006853 \approx 1$ respectively

From the results above, it shows that on average a young person can only visit any state once.

4.3 VARIANCE OF THE NUMBER OF VISITS

The variance is $\mathbf{Var}(f_{ij}) = \mathbf{F}(2\mathbf{F}_{dg} - \mathbf{I}) - \mathbf{F}_{sq}$

From $\mathbf{F}$

$$\mathbf{F}_{dg} = \begin{bmatrix} 1.018815 & 0 & 0 \\ 0 & 1.029291 & 0 \\ 0 & 0 & 1.006853 \end{bmatrix} \tag{4.7}$$

and

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4.8}$$

$$
\begin{aligned}
\mathbf{F}_{sq} &= \begin{bmatrix} 1.018815^2 & 0.821752^2 & 0.755628^2 \\ 0^2 & 1.029291^2 & 1.006853^2 \\ 0^2 & 0^2 & 1.006853^2 \end{bmatrix} \\
&= \begin{bmatrix} 1.037984 & 0.675276 & 0.570974 \\ 0 & 1.059440 & 1.013753 \\ 0 & 0 & 1.013753 \end{bmatrix} \\
2\mathbf{F}_{sq} &= \begin{bmatrix} 2.037630 & 0 & 0 \\ 0 & 2.058582 & 0 \\ 0 & 0 & 2.013736 \end{bmatrix} \\
2(\mathbf{F}_{dg} - \mathbf{I}) &= \begin{bmatrix} 1.03763 & 0 & 0 \\ 0 & 1.058582 & 0 \\ 0 & 0 & 1.013706 \end{bmatrix} \\
\mathbf{F}(2\mathbf{F}_{dg} - \mathbf{I} &= \begin{bmatrix} 1.018815 & 0.821752 & 0.755628 \\ 0 & 1.029291 & 1.006853 \\ 0 & 0 & 1.006853 \end{bmatrix} \begin{bmatrix} 1.03763 & 0 & 0 \\ 0 & 1.058582 & 0 \\ 0 & 0 & 1.013706 \end{bmatrix} \\
&= \begin{bmatrix} 1.057153 & 0.869892 & 0.765985 \\ 0 & 1.089589 & 1.020653 \\ 0 & 0 & 1.020653 \end{bmatrix}
\end{aligned}
\tag{4.9}
$$

$$\mathbf{Var}(f_{ij}) = \begin{bmatrix} 1.057153 & 0.869892 & 0.765985 \\ 0 & 1.089589 & 1.020653 \\ 0 & 0 & 1.020653 \end{bmatrix} - \begin{bmatrix} 1.037984 & 0.675276 & 0.570974 \\ 0 & 1.059440 & 1.013753 \\ 0 & 0 & 1.013753 \end{bmatrix}$$

$$= \begin{bmatrix} 0.019169 & 0.194616 & 0.195011 \\ 0 & 0.030149 & 0.006900 \\ 0 & 0 & 0.006900 \end{bmatrix} \tag{4.10}$$

The variances on young people who started from KCPE to KCPE, KCSE and college are 0.019169, 0.194616 and 0.195011 respectively. And their standard deviations are $0.138452 \approx 0$, $0.441553 \approx 0$ and $0.441600 \approx 0$. The variances on young people who started from KCSE to KCPE, KCSE and college are 0, 0.030149 and 0.0069 respectively. And their respective standard deviations are 0, $0.173635 \approx 0$ and $0.083066 \approx 0$. The variances on young people who started from College to KCPE, KCSE and college are 0, 0 and 0.0069 respectively. And their respective standard deviations are 0, 0 and 0.083066 ≈ 0. Since the standard deviations is zero, it means that deviations is insignificant.

4.3.1 TIME OF ABSORPTION - EXPECTED NUMBER OF STEPS

The expected number of times a young person is in any of the transient states (KCPE, KCSE and College) for a starting state s_i is the sum of all the entries in the ith row of $\mathbf{F}$.

$$
\mathbf{t} = \mathbf{Fc} = \begin{bmatrix} 1.018815 & 0.821752 & 0.755628 \\ 0 & 1.029291 & 1.006853 \\ 0 & 0 & 1.006853 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} 2.596195 \\ 2.036144 \\ 1.006853 \end{bmatrix}
$$

The expected number of times a young person is in KCPE, KCSE and College before career is $2.596195 \approx 3$, $2.036144 \approx 2$ and $1.006853 \approx 1$ respectively.

4.3.2 VARIANCE ON THE NUMBER OF STEPS

The variance on the number of steps before being absorbed when starting in a transient state i. The variance is

$$
\mathbf{t}_2 = (2\mathbf{F} - \mathbf{I})\mathbf{t} - \mathbf{t}_{sq}
$$

$$
\mathbf{t}_{sq} = \begin{bmatrix} 2.596195^2 \\ 2.036144^2 \\ 1.006853^2 \end{bmatrix} = \begin{bmatrix} 6.740228 \\ 4.145882 \\ 1.013753 \end{bmatrix}
$$

$$
2\mathbf{F} = \begin{bmatrix} 2.037630 & 1.643504 & 1.511256 \\ 0 & 2.058582 & 2.013706 \\ 0 & 0 & 2.013706 \end{bmatrix}
$$

$$
\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

$$2\mathbf{F}-\mathbf{I} = \begin{bmatrix} 2.037630 & 1.643504 & 1.511256 \\ 0 & 2.058582 & 2.013706 \\ 0 & 0 & 2.013706 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1.037630 & 1.643504 & 1.511256 \\ 0 & 1.058582 & 2.013706 \\ 0 & 0 & 1.013706 \end{bmatrix}$$

$$(2\mathbf{F}-\mathbf{I})\mathbf{t} = \begin{bmatrix} 1.037630 & 1.643504 & 1.511256 \\ 0 & 1.058582 & 2.013706 \\ 0 & 0 & 1.013706 \end{bmatrix} \begin{bmatrix} 2.596195 \\ 2.036144 \\ 1.006853 \end{bmatrix} = \begin{bmatrix} 7.561913 \\ 4.182931 \\ 1.020653 \end{bmatrix}$$

$$\mathbf{t}_2 = \begin{bmatrix} 7.561913 \\ 4.182931 \\ 1.020653 \end{bmatrix} - \begin{bmatrix} 6.740228 \\ 4.145882 \\ 1.013753 \end{bmatrix} = \begin{bmatrix} 0.821685 \\ 0.037049 \\ 0.006900 \end{bmatrix}$$

$\mathbf{t}_2$ is the variances associated with transition of young people among KCPE, KCSE and College which are 0.821685, 0.037049 and 0.006900. Their respective standard deviations are $0.906468 \approx 1$, $0.192481 \approx 0$ and $0.083066 \approx 0$. The deviations between KCPE and career is significant (that is approximately 1). This means that the expected number of times a KCPE graduate can be transient states before career is between 2 to 4. There is no significant change in the expected number of times a KCSE and College graduates be in transient states before they get into career.

4.4 CAREER

b_{ij} is the probability that an absorbing chain will be absorbed in the absorbing state s_j if it starts in the transient state s_i.

B is the matrix with entries b_{ij}. Then **B** is 3×1 matrix and

$$\mathbf{B} = \mathbf{FR} = (\mathbf{I} - \mathbf{Q})^{-1}\mathbf{R}$$

$$= \begin{bmatrix} 1.018815 & 0.821752 & 0.755628 \\ 0 & 1.029291 & 1.006853 \\ 0 & 0 & 1.006853 \end{bmatrix} \begin{bmatrix} 0.160627 \\ 0.104491 \\ 0.993194 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

B=FR gives the probabilities of a young person who was originally in KCPE, KCSE or College ends in career. **B** provides the long-run distribution of career of young people. From KCPE, KCSE and College, the probabilities of career are 1, 1 and 1 respectively. The value one shows that young people can start from different transient states(KCPE, KCSE and College), but finally end up in an absorbing state (Career).

Chapter 5

SUMMARY OF RESEARCH FINDINGS AND CONCLUSIONS

5.1 INTRODUCTION

The chapter gives a summary of discussions of the results, conclusions, recommendations and suggestions for farther research.

5.2 DISCUSSION OF THE RESULTS

About 16 percent (0.160627) of those who did KCPE, 10 percent (0.104491) of those who did KCSE and 99 percent (0.993194) college graduates got career. This shows that not many get into career after KCPE. Still fewer young people get into career after KCSE, this may be because most of them prefer to proceed to college before career.

A KCPE Graduate is expected to have taken $1.018815 \times 8 = 8.15052$ = 8 years in primary school, $0.821752 \times 4 = 3.27008$ years in secondary school and 0.755628×4 = 3.0225124 years in college before getting career. That is an average of 14 years before getting into career.

A KCSE Graduate is expected to take $1.029291 \times 4 = 4.117164$ years in secondary school and a further $1.006853 \times 4 = 4.027412$ years in college before getting career. That is an average of 8 years before getting into career. A College Graduate is expected to take $1.006853 \times 4 = 4.027412$ years in this state before getting career. That is an average of 4 years before getting into career.

The variances of KCPE with KCPE, KCPE with KCSE, and KCPE with college are 0.019169, 0.194616 and 0.195011 respectively. The standard deviations are $0.138452, 0.441153$ and 0.441600.

Variances of KCSE with KCPE, KCSE with KCSE, and KCSE with college are 0, 0.030149 and 0.0069 respectively. The standard deviations are $0, 0.173635$ and 0.083066.

Variances of College graduate with KCPE, College graduate with KCSE and College graduate with college graduate are 0, 0 and 0.0069 respectively. The standard deviations are $0, 0$ and 0.083066.Therefore, there is no much deviation from the expected. The variances associated with transition from KCPE, KCSE and College to career are 0.821685, 0.037049 and 0.0069 respectively. Their standard deviations are 0.906468 , 0.192481 and 0.083066. The low values indicate that the values did not deviate much from the expected, except for KCPE.

5.3 CONCLUSIONS

Not all who have career went through the three states; KCPE, KCSE and college. People got into career at different states. A higher state is not necessarily the only way to acquiring a career, though it has a very significant role into one having a career. Meanwhile, very many young people go into career after college compared to those who go for career with KCPE and KCSE Certificates. It shows that college level is very important regardless of one's academic level. It is clear that chances of one getting into career is more certain even if they didn't do what they were trained for.

5.4 RECOMMENDATIONS

Young people should be encouraged to have career in mind as early as when they are in primary and study towards that. They should either stop at primary or go through to college to increase their chances of getting a career.

5.5 SUGGESTION FOR FURTHER RESEARCH

- The study should be done with a bigger sample in different parts of the country.

- The people with career to be categorized into white collar and blue collar jobs.

- Consideration of those who take a shorter or a longer duration in every state.

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