A STOCHASTIC AND DETERMINISTIC PREDATOR-PREY POPULATION MODEL FOR SUSTAINABLE HARVESTING:

A CASE OF NILE PERCH *Lates niloticus* AND NILE TILAPIA *Oreochromis niloticus*

By

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A thesis submitted in partial fulfilment of the requirement for the award of the degree of

Master of Science in Applied Statistics

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ABSTRACT

Predation by Nile perch is one of the main causes of fish stock depletion in Lake Victoria. Uncontrolled human exploitation (overfishing) of the stock leads to loss of fish biodiversity in Lake Victoria. Increasing the harvesting rates of mature Nile perch can lead to stable stationary states. However, increasing the predation rates of Nile perch can lead to unstable ecosystem. Predation, coupled with uncontrolled harvesting have been a major ecological force in shaping the present day fishing communities on Lake Victoria, in particular and other lakes at large. The challenges currently facing Lake Victoria is how to sustain the Nile perch, Nile tilapia and how to conserve and restore threatened fish species. In this study, we will apply the Lotka Volterra competition model (1925) for predator and prey population.

The objective of this study is therefore to develop a predator- prey model for Nile perch and Nile tilapia based on standard Lotka- Volterra predator prey model. The methodology involves formulation of a system of ordinary differential equations where prey ratio is incorporated. in this model. The finding in this study could be used to predict and explain the effect of predation by Nile perch on tilapia so that control of Nile perch can result in conservation and restoration of threatened fish species. The model is also a contribution of knowledge in mathematical modeling.

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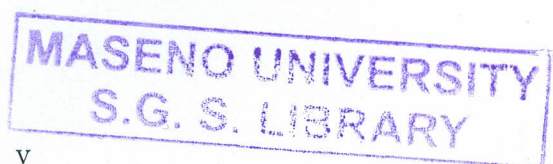
Chapter 1

INTRODUCTION

1.1 Introduction

From ancient times, fishing has been a major source of food for humanity
and a provider of employment and economic benefits to those engaged in
this activity. However, with increased knowledge and the dynamic devel-
opment of fisheries, it was realized that living aquatic resources, although
renewable, are not infinite and need to be properly managed, if their con-
tribution to the nutritional, economic and social well-being of the growing
world's population was to be sustained.

Kenya's fisheries sub-sector is based on three main fish resource bases,
namely: inland fresh-water, coastal marine and aquaculture. Of these,
inland fresh-water fisheries are the most important with Lake Victoria
dominating fish production. The lake alone has contributed 92 percent
(equivalent to 142000 tones) of an annual mean of 155000 tonnes of fish
landed in Kenya between 1998 and 2005. Besides Lake Victoria, the other
fresh water fish sources are lakes Turkana, Baringo, Naivasha, Jipe and
several dams and rivers spread across the country which collectively pro-

duce 3 percent of total fish. Marine and aquaculture fisheries constitute only about 4 and 1 percent respectively of fish landed in the country. Three species- Nile perch (*Lates niloticus*), Dagaa (*Rastrineobola argentia*) and Tilapia (*Oreochromis niloticus*) - constituted 52, 33, and 10 percent respectively of the total fish caught in Lake Victoria, with all the other species contributing just about 5 percent. Abila [1] cited that the indiginous fish species had been overtaken by exotic species in fish production. Before the introduction of Nile perch and exotic cichlids in the 1950's, the tilapiines: *Oreochromis esculentis* (Graham) and *Oreochromis variabilis* were the most important commercially.

The Lake Victoria fish stock has undergone remarkable changes over the past 20 years. Signs of over fishing were reported as early as 1970 when the catch rates of tilapia dropped from 50-100 fish per 50 m long net with 127 mm stretched mesh to less than 5 fish (Ssetongo 1972). As the stocks of Nile perch increased, fish species diversity decreased rapidly. Many important food fish species seemed to have disappeared and later even *L. niloticus* showed signs of decline.

From the conservation point of view, the decline and virtual extinction of the cichlid species must be considered as an irreplaceable loss. It is anticipated that one of the reasons behind the decline in the fish species diversity is predation. Other reasons, according to Lande[13] could be over fishing and environmental changes. The indigenous tilipiines were eliminated as a result of inter-specific species competition with introduced exotic species, mainly *O. niloticus*.

According to Ogutu[24], Lake Victoria evolved into a fishery dominated by three species; the two exotic species- *Lates niloticus* (Nile perch), *Ore-*

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ochromis niloticus (Nile tilapia), and one native species- *Rastrineobola argentia*(Dagaa).

The challenge facing us today is the consideration of global change, the loss of biodiversity, and achieving a sustainable future, elevating the complexities to new levels. It is clearly necessary to develop an ecologically acceptable strategy for harvesting any renewable resource, be it animals, fish or plants. Murray[21] observed that one of the basic concepts in the analysis of harvesting populations is that of Maximum Sustainable Yield (MSY) with minimum effort.

1.2 Basic concepts and definitions

1.2.1 Stochastic processes:

A stochastic process is a family of random variables, $\{X(t) : t \in T\}$,where t denotes time. That is, at every time t in set T, a random number X(t) is observed. $\{X(t) : t \in T\}$ is a discrete- time process if set T is finite or countable[3].

In practice, this generally means T= $\{0,1,2,3,...\}$.

Thus a discrete- time process is $\{X(0), X(1), X(2), X(3),...\}$.

$\{X(t) : t \in T\}$ is a continuous- time process if T is not finite or not countable i.e. if the times form a continuum.

In practice, this generally means $T = [0, \infty)$ or $T = [0, K]$ for some K.A continuous- time process $\{X(t) : t \in T\}$ has a random number X(t) recorded at every instant time. The state space, S is the set of real values that X(t) can take. Every X(t) takes a value in $\mathbb{R}$. but S will often be a

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smaller set: $\mathbb{R}$. For example , if X(t) is the outcome of a coin tossed at time t, then the state space S= {0,1}.

The state space S is discrete if it is finite or countable. Otherwise it is continuous. The state space S is the set of states that a stochastic process can be in Feller[7].

A stochastic process is a variable that evolves over time in a way that is at least in part random. In most cases, a stochastic variable has both an expected value term (drift term) and a random term (diffusion term). The drift coefficient models the dominant action of the system while the diffusion coefficient represents randomness along the dominant curve. Fish population growth varies in a random manner. This is because various phenomena which are unpredictable affect it. Such phenomena include disease, and environmental fluctuations which may harm or destroy the fisheries. Fish population growth therefore represents a stochastic process.

1.2.2 Markov process

A Markov process is a particular type of stochastic process where the future distribution is dependent only on the current value. The earlier history of the variable and the way in which the current value emerged from the past are irrelevant as far as prediction using Markov models are concerned.

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1.3   Birth-death process

The Birth- death process is a special case of a continuous- time Markov process where the states represent the current size of a population and where the transitions are limited to births and deaths. Birth- death process have many applications in demography, queuing theory, or in Biology, for example in the evolution of bacteria.

When a birth occurs, the process goes from state n to state n+1.

The process is specified by the birth rates $\{\lambda_i\}$, $i = 0,....\infty$ and death rates $\{\mu_i\}$, $i = 0,...\infty$.

In birth- death processes, state changes can only happen between neighbours.

- Size of population:

- system is in state E_k when it consists of k members

- changes in a population occur by at most one.

- size increased by one $\longrightarrow$ "Birth"

- size decreased by one $\longrightarrow$ "Death"

Transition probabilities P_{ij} do not change with time.

$$P_{ij} = \begin{cases} \mu_i & j = i - 1 \\ 1 - \lambda_i - \mu_i & j = i \\ \lambda_i & j = i + 1 \\ 0 & otherwise \end{cases}$$

- μ_i= death (less one in population size)

- $\mu_0 = $ (no population $\longrightarrow$ no death)

- λ_i= birth (increase one in population)

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• $\lambda_i > 0$ (birth is allowed)

<u>Queuing Theory Model</u>

• Population = customers in the queuing system

• Death = a customer departure from the system

• Birth = a customer arrival in the system

1.4   Pure birth process

Suppose that a population develops over a short period of time in crowd-free conditions with no environmental resistance factors, assuming that:

• The organisms do not die.

• They develop without interacting with each other.

• The birth rate λ is same for all organisms.

  $\mu_k{=}0$ for all k

  $\lambda_k = \lambda$ for all k

  The system begins at time t_0 with 0 member.

<u>Deterministic Model</u>

Letting N(t) denote population size at time t. Then in subsequent time interval of length h, the increase in population size due to a single organism is $\lambda \times h.(i.e.rate \times time)$. so that increase in size due to all N(t) organisms is $\lambda \times h \times N(t)$.

Thus

$$N(t + h) = N(t) + \lambda h N(t).$$

Then on dividing both sides by h gives:

$$[N(t+h) - N(t)] \div h = \lambda N(t)$$

Letting h approach zero then yields the differential equation

$$\frac{dN(t)}{dt} = \lambda N(t).$$

which integrates to give

$$N(t) = N(0) \exp(\lambda t)$$

where N(0) denotes the initial population size at time $t = 0$. This form for N(t) is known as the Malthusian expression for population development. Taking logs of both sides gives

$$\ln[N(t)] = \ln[N(0)] + \lambda t$$

Stochastic model

The above model is purely deterministic since it assumes that each organism reproduces on a completely predictable basis at a constant rate. In reality however, population growth is 'stochastic' (i.e. random).

Suppose that in a short time interval of length h, the probability that birth will occur is λh. Then for the population to be of size N at time t and no birth occurs in the subsequent short time interval (t , t + h), or else it is of size N - 1 at time t and exactly one birth occurs in (t , t + h). By choosing h sufficiently small, we may ensure that the probability

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of more than one birth occurring is negligible. Since the probability of N increasing to N+1 in (t, t + h) is $(\lambda h) \times N$. It follows the probability of no increase in (t, t + h) is $1 - \lambda N h$. Similarly, the probability of N - 1 increasing to N in (t, t + h) is $\lambda(N - 1)h$

Thus on denoting

$P_N(t)=$ Pr(population is of size N at time t).

we have

$P_N(t + h) = P_N(t) \times \text{Pr}\{$ no birth in (t, t+h)$\}$+ $P_{N-1}(t) \times$ Pr$\{$ one birth in (t, t + h)$\}$.

i.e. $P_N(t + h) = P_N(t) \times (1 - \lambda N h) + P_{N-1}(t) \times \lambda(N - 1)h$

on dividing both sides by h, we get:

$$[P_N(t + h) - P_N(t)] \div h = -\lambda N P_N(t) + \lambda(N - 1)P_{N-1}(t)$$

as h approaches zero, this becomes

$$\frac{dP_N(t)}{dt} = -\lambda N P_N(t) + \lambda(N - 1)P_{N-1}(t)$$

For N(0), N(0) + 1,...

whose solution is given by the negative binomial distribution:

$$P_N = \binom{N - 1}{n_0 - 1} e^{-\lambda n_0 t}(1 - e^{-\lambda t})^{N - n_0}$$

$(N = n_0, n_0 + 1, ...)$

1.5 Pure death process

Suppose an environment of an Isolated population is polluted to such an extent that all future reproduction is prevented and the death rate of individual members is independent of their age, then a pure death process will result.

Assuming that:

- Organisms do not give birth.

- They develop completely independently from each other

- The death rate μ is the same for all individuals and does not change with time.

- The individuals do not age

$\mu_k = \mu \geq 0$ for all k

$\lambda_k = 0$ for all k

The system begins with N members, k= 1,2,3,...N

Deterministic model

Suppose that in a small time interval of length h the decrease in population size due to a single organism is $\mu \times h$. Then the decrease in size due to all N(t) organisms is $\mu \times h \times N(t)$. Thus

$$N(t + h) = N(t) - \mu h N(t)$$

which on paralleling with pure birth argument, becomes:

$$\frac{dN(t)}{dt} = -\mu N(t)$$

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with the solution

$$N(t) = N(0)\exp(-\mu t).$$

Taking logarithms gives the linear relationship

$$\ln\{N(t)\} = \ln\{N(0)\} - \mu t$$

Stochastic model

Denote q(t) = Pr(a particular organism is alive at time t)

then

q(t + h) = Pr (it is alive at time t and does not die in the subsequent small time interval h)

$$= q(t) \times (1 - \mu h)$$

On letting h approach zero yields

$$\frac{dq(t)}{dt} = -\mu q(t)$$

which integrates directly to give

$$q(t) = \exp(-\mu t)$$

hence

p(t) = Pr (the organism is dead by time t)

$$= 1 - q(t) = 1 - e^{-\mu t}.$$

Thus if the initial population is of size $N(0) = n_0$, and if all organisms behave independently of each other, then N(t) satisfies the conditions for

the binomial distribution with probabilities

$$P_N(t) = \binom{n_0}{N} [q(t)]^N [p(t)]^{n_0 - N}$$

i.e.

$$P_N(t) = \binom{n_0}{N} e^{-N\mu t}(1 - e^{-\mu t})^{n_0 - N} \quad (N = 0, 1, \ldots n_0)$$

The mean and variance are given by the results $n_0 q(t)$ and $n_0 q(t) p(t)$

i.e. $m(t) = n_0 e^{-\mu t}$ and

$v(t) = n_0 e^{-\mu t}(1 - e^{-\mu t})$.

1.6   Literature Review

A number of scholars have studied predator- prey models.
Lotka[18] modeled deterministic predator- prey relationship using the following differential equations:

$$\frac{dN(t)}{dt} = N(\Lambda - \nu R)$$

$$\frac{dR(t)}{dt} = R(-\mu + \lambda N)$$

where N(t) represents number of prey at time t, R(t) represents number of predators at time t, ν represents death rate of prey and λ represents birth rate of predator. It is assumed that in absence of predators, prey increase at rate Λ, while in absence of prey, predators die at rate μ In a later paper, he introduced a logistic term $-cN^2$ into the prey model and came up with the model:

$$\frac{dN}{dt} = N(\Lambda - cN - \nu R)$$

$$\frac{dR}{dt} = R(-\mu + \lambda N)$$

Even with the introduction of the logistic term $-cN^2$ the model did not improve much since it only strengthened the death term already present. In his model, he assumed the following:

(i) in the absence of predators, the prey population develops as a logistic process with intrinsic rate of increase Λ and carrying capacity $\frac{\Lambda}{c}$.

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(ii) the rate at which prey are eaten is proportional to the product of the two population sizes.

(iii) the rate at which predators are born is proportional to the product of the two population sizes and no time lag is involved.

Leslie and Gower [16], retained the Volterra prey equation:

$$\frac{dN}{dt} = N(\Lambda - cN - \nu R)$$

but changed the character of the predator equation to:

$$\frac{dR}{dt} = R(\mu - \lambda(\frac{R}{N}))$$

In this model, the net predator growth rate takes account of the relative sizes of the two populations. The larger $\frac{R}{N}$ becomes, the smaller the number of prey available to each predator and consequently the resource available for predator growth declines.

Ogana et al[22] have described aquatic ecosystems as being complex webs of interactions between many species, and between these species, the environment and man. Factors limiting fish populations could thus be physical, based on meteorological elements such as temperature and precipitation or biological, such as reproduction, predation and competition, or energetic such as food chain nutrient transfers. The above processes coupled with many other ecological and environmental factors make it difficult to accurately asses the impact of any one of them on marine and

fresh water fisheries.

Concern over predation, harvesting and competition of species in ecology has stimulated the development of several mathematical models to help understand and explain the population dynamics of interacting species. Gaucel *et al*[9] developed mathematical models to explain invading introduced species in insular heterogeneous environments. In their model, they emphasize the importance of accurate estimates of the predation rate on the different age and sex classes as well as on demographic parameters of prey populations in determining those species that need to be protected. Gaucel and Pontier[9] used mathematical modeling to explain how predator food preference can change the destiny of native prey in predator- prey systems. The predator- prey system behavior when the predator population has a strong preference for one of the two age stages of the prey population was described in the model. They showed how the age structure in the prey population can modify the dynamics of the population under study. The preying preference of the alien predator on either juvenile or adult stages of the mature prey population, affects the dynamical behavior of both native and introduced population densities.

Watsala[33] developed a mathematical model to explain the changes in some fish and invertebrates population in Lake Victoria in relation to predation by Nile perch and human exploitation. In Watsala's model, the prey fish species comprised of Haplochromine cichlids (*Rastrineobola argentea , Oreochromis niloticus*) and Juvenile Nile perch while the prey of the invertebrates were *Cardina nilotica* and *Anisopteran* nymphs. The Nile perch population in Watsala's model was divided into five developmental stages which included: eggs(Larvae), young, juvenile, sub-adult

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and adults. The four stages of the Nile perch were graded on the basis of the length of the Nile perch. The model further assumes the Nile perch to be a cannibal and cannibalism only affected the juvenile Nile perch stage. Watsala[33] assumed the prey behavior to be the same in various developmental stages. The model does not put into consideration the developmental stages of the prey species. The age of the prey (e.g. the Nile tilapia) is of great significance as it determines the predation and harvesting impact and response e.g. mature Nile tilapia (*Oreochromis niloticus*) has the ability of avoiding predators and fishing nets compared to the young ones.

Mugisha *et al*[20] came up with a mathematical model that incorporated three developmental stages (i.e. young, juvenile and mature) of the Nile perch and Nile tilapia with the behavior of the prey towards predation and harvesting being formulated.

In this study, we wish to incorporate the proportion function to modeling the proportion of Nile perch.

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1.7 Statement of the problem

In this study, we shall to incorporate predator- prey ratio in the Lotka-Volterra model and develop a predator- prey model that can be used to predict and explain the effect of predation by Nile perch on tilapia.
We shall also investigate the effect of harvesting of both Nile perch and tilapia on the population dynamics.

1.8 Objectives of the study

The main objective of this study is to develop a predator- prey model that incorporates the predator-prey ratio. To this end the specific objectives of the study will be to:

(a) Review the existing predator- prey models

(b) Develop a predator- prey model that incorporates the predator- prey ratio in its formulation

(c) To fit the developed model in (b) to simulated data using parameters estimated from previous studies on the two species of fish namely, Nile perch and Nile tilapia.

(d) Discuss the implications of varying the predator- prey ratio on the sustainability of the population sizes of the two species of fish.

1.9   Significance of the study

The importance of fishery resources to the economy of Kenya cannot be understated. Fish make a significant contribution to the Gross Domestic Product, provides both direct and indirect employment and supply relatively cheap protein to the population. This role of aquatic products is still rising, because land for agriculture has been overexploited to a large extent and no new areas for agriculture can be found because of limitation of land.

It is hoped that the model could be used to predict and explain the effect of predation by Nile perch on tilapia. This information is expected to assist interested parties to make informed decisions about management of fisheries. It is also hoped that it can be extended to similar situations of over exploitation of other renewable resources with an aim of conservation and restoration of threatened species.

Chapter 2

POPULATION MODELS

2.1 Introduction

Here, we consider a model in which the proportion of prey has been incorporated. This model can be used to set the prey proportion to an expected ratio to prevent extreme fluctuation that can cause imbalance in the ecosystem.

Assumptions

The following assumptions are made when constructing the model:

(a) Preference in the predation nature of Nile perch is assumed.

(b) Cannibalism by Nile perch is negligible.

(c) Prey and predator can die naturally.

(d) Because the species are in a vast lake, the system is assumed to be at a steady state.

We will also use the following notations:

R- density of Tilapia

P- density of Nile perch

$R(t)$- size of tilapia population at time t.

$P(t)$- size of Nile perch population at time t.

$N(t)$ -total number of Tilapia and Nile perch at time t.

The following parameters are used in the model:

Λ- birth rate of tilapia

λ- birth rate of Nile perch

ν-death rate of tilapia

μ- death rate of Nile perch

$\phi(t) = \frac{R(t)}{N(t)}$

$\hbar_1$- harvest rate of prey (catchability parameter)

$\hbar_2$- harvest rate of predator (catchability parameter)

2.2 Simple predator- prey model:

The model below is a simple form in which the within species competition has been ignored:

$$\frac{dR(t)}{dt} = R(t)(\Lambda - \nu P(t)) \tag{2.1}$$

$$\frac{dP(t)}{dt} = P(t)(-\mu + \lambda R(t)) \tag{2.2}$$

The constant ν measures the death rate of prey due to being eaten by predators; the greater the number of predators, the faster the prey population will be depleted. The constant λ measures the skill of predator in catching prey; the greater the number of prey, the greater the availability of predator food resource.

We combine the equations (2.1) and (2.2) to get:

$$\frac{dR(t)}{dP(t)} = \frac{R(t)(\Lambda - \nu P(t))}{P(t)(-\mu + \lambda R(t))} \tag{2.3}$$

which we can write in the form:

$$[-(\mu/R(t)) + \lambda]dR(t) = [(\Lambda/P(t)) - \nu]dP(t) \tag{2.4}$$

which integrates directly to:

$$\mu \ln R(t) - \lambda R(t) + \Lambda \ln P(t) - \nu P(t) = c \tag{2.5}$$

where c is a constant. This expression represents a family of closed curves in which each member of the family correspond to a different value of the constant. Each curve is determined by the initial position (R(t)(0), P(t)(0)).

Five such curves are illustrated in figure 2.1 for the process:

$$\frac{dR(t)}{dt} = R(t)(1.50 - 0.1P(t)) \tag{2.6}$$

$$\frac{dP(t)}{dt} = P(t)(-0.25 + 0.01R(t)) \tag{2.7}$$

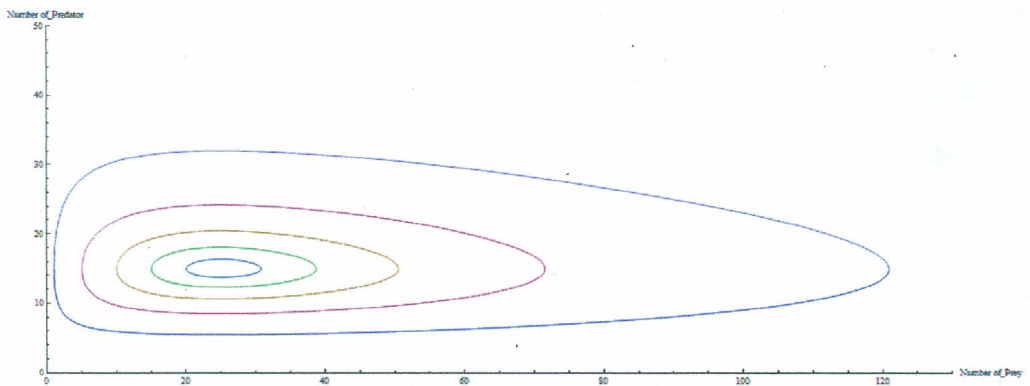with start points P(0)=15 and R(0)=1, 5, 10, 15 and 20

Figure 2.1: Family of closed curves for the Lotka- Volterra process

Dynamics of the system

In the model system, the predators thrive where there are plentiful of prey, but ultimately outstrip their food supply and decline. As the predator population is low, the prey population will increase again. These dynamics continue in a cycle of growth and decline.

Population equilibrium

Population equilibrium occurs in the model when neither the population levels is changing. When both derivatives are equal to 0.

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$R(t)(\Lambda - \nu P(t)) = 0$

$-P(t)(\mu - \lambda R(t)) = 0$

When solved for R(t) and P(t) the above system of equations yields:

$(P(t) = 0, R(t) = 0)$

and

$(P(t) = \frac{\Lambda}{\nu}, R(t) = \frac{\mu}{\lambda})$

Hence, there are two equilibria.

The first solution effectively represents the extinction of both species. If both populations are at 0, then they will continue to be so indefinitely. The second solution represents a fixed point at which both populations sustain their current, non-zero numbers, and in the simplified model, so indefinitely. The levels of population at which this equilibrium is achieved depend on the chosen values of the parameters $\Lambda, \mu, \nu, and \lambda$.

2.3 Population model with proportion of prey incorporated

Suppose we wish to incorporate a proportion of prey in the model:

Let

$$\phi(t) = \frac{R(t)}{N(t)} \tag{2.8}$$

Where

$$N(t) = R(t) + P(t)$$

and therefore

$$\frac{P(t)}{N(t)} = 1 - \phi(t)$$

$$\frac{d\phi(t)}{dt} = \frac{d}{dt}\left(\frac{R(t)}{N(t)}\right)$$

$$\frac{d\phi(t)}{dt} = \frac{N(t)\frac{dR(t)}{dt} - R(t)\frac{dN(t)}{dt}}{(N(t))^2} \tag{2.9}$$

$$= \frac{1}{N(t)}\frac{dR(t)}{dt} - \frac{R(t)}{(N(t))^2}\frac{dN(t)}{dt}$$

$$= \frac{R(t)}{N(t)}\left[\frac{1}{R(t)}\frac{dR(t)}{dt} - \frac{1}{N(t)}\frac{dN(t)}{dt}\right]$$

$$= \phi(t)\left[\frac{1}{R(t)}\Big(R(t)(\Lambda - \nu P(t))\Big) - \frac{1}{N(t)}(\lambda - \mu)N(t)\right]$$

$$= \phi(t)\Big[\Lambda - \nu P(t) - (\lambda - \mu)\Big] \tag{2.10}$$

$$\frac{d\phi(t)}{dt} = \phi(t)[\Lambda - \nu P(t) - (\lambda - \mu)]$$

$$\implies \frac{d\phi(t)}{\phi(t)} = [\Lambda - \nu P(t) - (\lambda - \mu)]dt$$

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$$\int \frac{d\phi(t)}{\phi(t)} = \int [\Lambda - \nu P(t) - (\lambda - \mu)]dt \qquad (2.11)$$

$$\ln(\phi(t)) = [\Lambda - (\lambda - \mu)]t - \nu \int P(t)dt + c$$

$$\implies \phi(t) = e^{[\Lambda-(\lambda-\mu)]t-\nu \int P(t)dt} \cdot e^c$$

Let

$$e^c = c_\circ$$

(initial ratio as a result of initial population of both fish type)

$$\phi(t) = c_\circ \cdot e^{[\Lambda-(\lambda-\mu)]t} \cdot e^{-\nu \int P(t)dt} \qquad (2.12)$$

At carrying capacity

$$P(t) = \frac{\Lambda}{\nu}$$

we get:

$$\phi(t) = c_0 e^{[\Lambda-(\lambda-\mu)]t} \cdot e^{-\nu \int \frac{\Lambda}{\nu}}$$

$$\phi(t) = e^{(\Lambda-(\lambda-\mu))t} \cdot e^{-\Lambda t}$$

$$\phi(t) = e^{-(\lambda-\mu)t} \cdot e^c \qquad (2.13)$$

Chapter 3

EMPIRICAL STUDY

In this chapter, we present numerical data in the model. The parameter values used are estimated from the literature of various research articles on Lake Victoria as follows:

3.1 Mortality rates

Two forms of mortality rate are considered for this research: fishing mortality caused by harvesting of the species and natural mortality due to natural causes (diseases, predation, and water pollution). The two forms combined lead to total mortality(Z) of the species. In fisheries,

Total mortality(Z)= Fishing mortality ($\hbar$) + Natural mortality (μ)

3.1.1 Natural mortality

Bassa [4] estimated the total mortality (Z) of Nile perch and Nile tilapia as $1.65yr^{-1}$ and $0.74yr^{-1}$ respectively. Natural mortality rates μ for the Nile perch and ν for the Nile tilapia were calculated and found to be

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$0.34yr^{-1}$ and $0.18yr^{-1}$ respectively. The life expectancy of the Nile perch is estimated to be 2 years and 10 months. Thus we"ll consider the natural mortality μ to be $0.35yr^{-1}$ which is the reciprocal of life expectancy. Watsala [33] estimated mortality rates of Nile perch to be ranging from 0.221 for the young ones to 0.57 for the mature species of 7 years and above.

Dache [6] studied the landing trends, growth and mortality rates of Nile tilapia (*Orechromis niloticus*) to asses the state of its stock in Lake Victoria. Estimated total, natural and fishing mortality rates where $0.71yr^{-1}$, $0.72yr^{-1}$ and $0.99yr^{-1}$ respectively. Life expectancy of the Nile tilapia is estimated to be 1 year and 5 months. Fishing mortality of the Nile tilapia $\hbar_1$ is estimated at $0.468yr^{-1}$.

3.1.2   Fishing mortality

Rabuor *et al*[27] estimated total mortality rates (Z) of Nile perch from the catch curve analysis of two sets of length frequency data. The estimated rates were $0.72yr^{-1}$ and $0.94yr^{-1}$. For this study, we consider $Z = 0.83yr^{-1}$ which is the average of Rabuor's estimates. Thus from $\hbar_2 = Z - \mu$, we have $\hbar_2 = 0.48yr^{-1}$.

Getabu [10] analysed length frequency data to study mortality, exploitation rate and recruitment in Nile tilapia (*oreochromis niloticus*) in Nyanza Gulf, Lake Victoria. The estimated fishing mortality ($\hbar_1$) was $0.468yr^{-1}$. Because similar harvesting methods are still being practised on Lake Victoria, we consider $\hbar_1 = 0.468yr^{-1}$ for this paper.

Fishing mortality rate $\hbar_1$ for tilapia and $\hbar_2$ for Nile perch can be esti-

mated from the catches in the appendix.

The birth rates of Nile perch and Nile tilapia are $\Lambda = 1.0123$ and $\lambda = 0.910$ per year respectively.

3.2 Numerical simulation of the model

The parameters used for simulation are taken from published records. These will be used to generate curves for the model that we have come up with. $\mu = 0.34yr^{-1}$

$\nu = 0.18yr^{-1}$

$\lambda = 0.910yr^{-1}$

$\Lambda = 1.0123yr^{-1}$

$\phi(t) = \frac{1}{2}e^{-(\lambda-\mu)t}$

$\phi(0) = 0.5$

$\phi(1) = 0.2827$

$\phi(2) = 0.1599$

$\phi(3) = 0.0904$

$\phi(4) = 0.0511$

$\phi(20) = 0.00000598$

When $\phi(t)$ is plotted against time, the following curve is obtained:
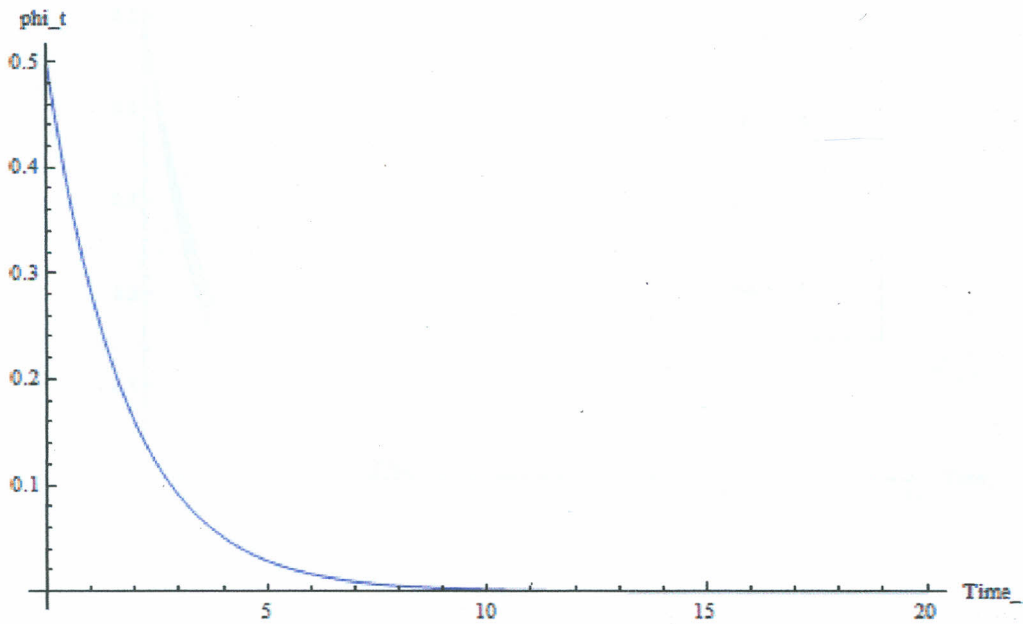
Figure 3.1: Mortality of 0.34 used at equilibrium

At t=0, $\phi(0) = 0.5$ it decreases to 0.0904 at t=3 and continues decreasing gradually and finally becomes extinct.

Suppose we vary the death rate of predator, say by increasing its harvesting rate. The result would be that we are merely delaying the extinction time as depicted by the following figure:
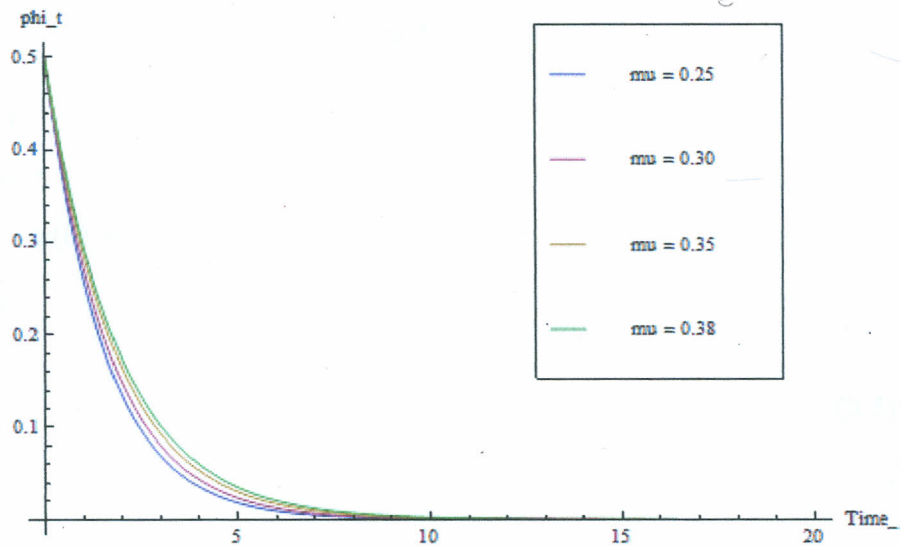
Figure 3.2: Influence of mortality on phi(t)

3.2.1 Predator population constant

When predator population is maintained relatively constant by making birth rate equal to death rate: From equation (2.10) we have: $\phi(t) = e^{[\Lambda-(\lambda-\mu)]t}.e^{-\nu \int P(t)dt}$ Assume $P(t) = c$, i.e. population of Nile perch is constant c:

$$\phi(t) = e^{[\Lambda-(\lambda-\mu)]t}.e^{-\nu ct+k} \qquad (3.1)$$

$\phi(t) = e^{[\Lambda-(\lambda-\mu)]t}.e^{-\nu ct}.e^{k}$

Let $e^{k} = A$

$\phi(t) = Ae^{[\Lambda-(\lambda-\mu)]t}.e^{-\nu ct}$

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$$\phi(t) = Ae^{[\Lambda-\lambda+\mu-\nu c]t} \tag{3.2}$$

at t= 0

$\phi(0) = A$

assume $\phi(0) = \frac{1}{2}$ initial ratio

$A = \frac{1}{2}$

$$\phi(t) = \frac{1}{2}e^{[\Lambda-\lambda+\mu-\nu c]t} \tag{3.3}$$

To achieve $\frac{1}{3}$ at t= 10 , c will be 2.6863 To achieve $\frac{1}{3}$ at t= 100 , c will be 2.4836 To achieve $\frac{1}{3}$ at t= 1000 , c will be 2.4634 By maintaining P(t) at 2.4634:

at t=0, $\phi(t) = 0.5$

at t=10, $\phi(t) = 0.4979$

at t=1000, $\phi(t) = 0.3312$
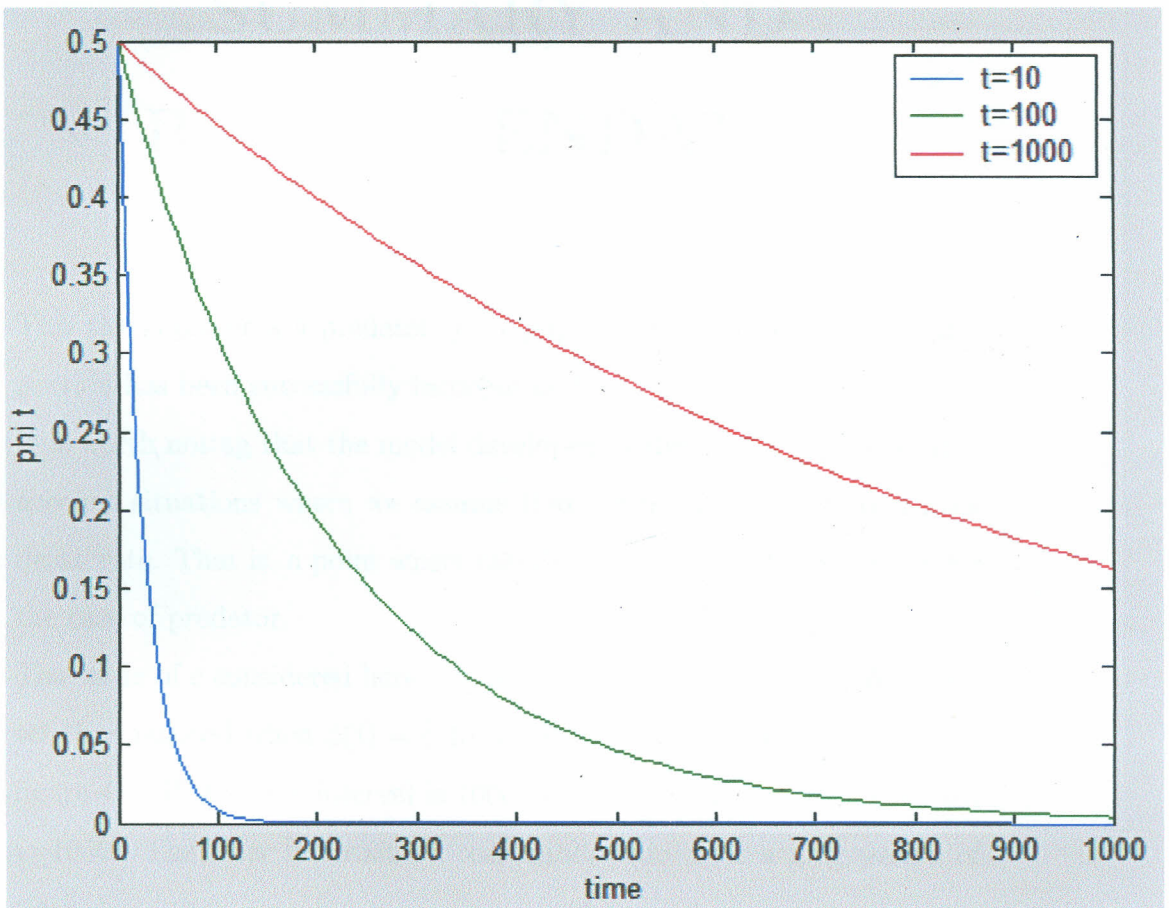
at t=10000, $\phi(t) = 0.00812$

Figure 3.3: Time specific curves with P(t) maintained constant

Chapter 4

SUMMARY AND
RECOMMENDATON

This thesis presents a predator- prey population model where prey proportion has been successfully incorporated.

It is worth noting that the model developed in this study only takes into account situations where we assume that birth rate of predator equals death rate. That is, a point where carrying capacity has been reached for the case of predator.

The value of c considered here is 2.4634. This is taken as P(t). When we set time interval when $\phi(t) = \frac{1}{3}$ to be 10 years, we obtain curve t=10 in figure 4.3. If the time interval is 1000 years, we obtain the curve labeled t=1000. The later maintains a reasonable ratio after a long period of time.

The value 2.4634 considered is multiplied by 10^5 and is a measure in metric tonnes.

There are other issues that this study was unable to address due to its scope. In view of this, the following are recommended for further research:

(1) Nile Tilapia population depends on several ecological conditions like food supply, prevailing temperature etc. A population model that takes into account these factors should be considered.

(2) Future models developed should take into consideration variations in the birth rate and death rate, rather than assuming that all the rates are the same for all intervals.

(3) Future models should also consider the effect of migration into or out of the population as this would affect the population of both predator and prey.

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